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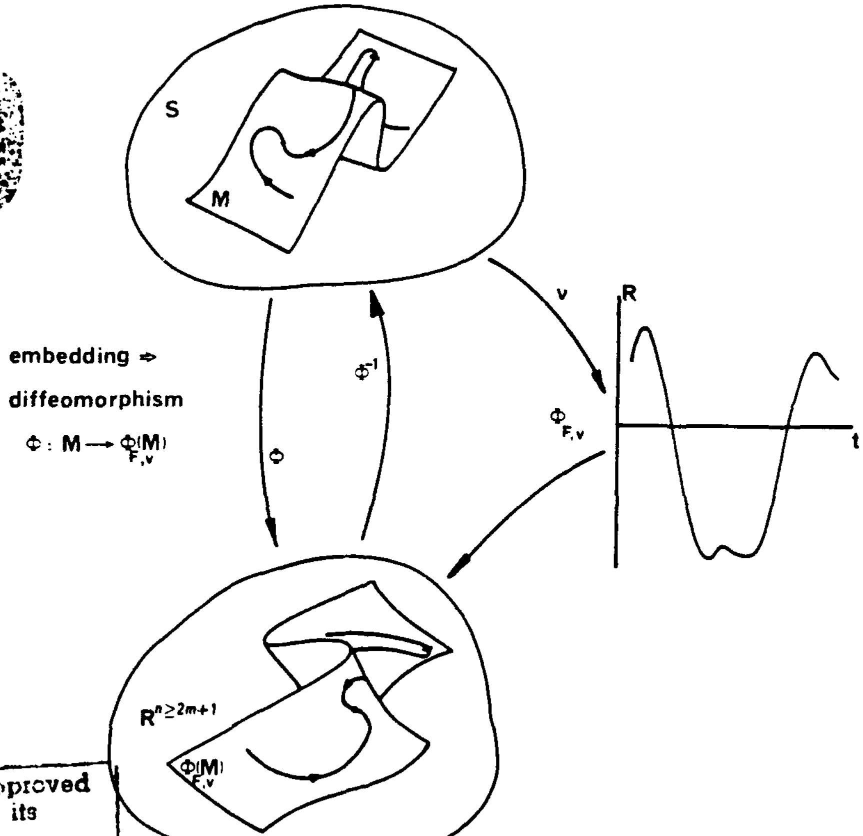
IUTAM SYMPOSIUM AND NATO ADVANCED RESEARCH WORKSHOP

on

Interpretation of Time Series from Nonlinear Mechanical Systems

26-30th August, 1991
University of Warwick, Coventry, England

CONFERENCE ABSTRACTS



A. LECTURES

B. POSTERS

We would like to acknowledge the generous support from: ***Royal Society of London, London Mathematical Society, British Protroleum Plc, U.S. Army, U.S. Air Force.***

92-02449

Programme

All lectures will be in room ELT 1, Mathematics Institute

Monday 26 August

9:00	*Welcome*	
	Coherent Structures and Turbulence	
9:10	Lumley	*Low dimensional model of the wall region in a turbulent.boundary layer: new results*
10:00	Chauve	*Applications of the bi-orthogonal decomposition in fluid mechanical experiments*
10:25	*Coffee Break*	*All coffee and tea breaks will be in the Graduate Common Room, Mathematics Institute*
10:50	Sato	*Objective search of patterns in turbulence signals*
11:15	Narasimha	*A simple dynamical system that mimics open-flow ...*
11:40	Levchenko	*Flow randomization in boundary-layer transition*
12:05	Ciliberto	*Estimating the number of degrees of freedom in spatially extended systems*
12:30	*Lunch Break*	*Rootes Hall Refectory*
2:00	Poster Session A & Discussion	
3:30	*Tea Break*	
	Prediction and noise I	
4:00	Theiler	*Testing for nonlinearity in time series: the method of surrogate data*
4:25	Smith	*Identification and prediction of chaotic dynamical systems*
4:50	Provenzale	*Deterministic chaos versus random noise: finite correlation dimension and converging K2 entropy for colored noises with power-law spectra*
5:15	*Brain Break*	
	Symmetry	
5:30	Stewart	*Symmetric attractors and chaotic time series*
5:55	Nicolaenko	*Detecting symmetry breaking intermittent chaos in turbulence*
6:20	END of DAY	
7:12	*Dinner*	*Rootes Hall Refectory*

Tuesday 27 August

9:00	*Announcements*	
	Wavelets	
9:10	Arneodo	*Wavelet analysis of fractal signals*
10:00	Grossmann	*Continuous wavelet transform*
10:25	*Coffee Break*	
	Characterizing Turbulence	
10:50	Ching	*Transitions in convective turbulence: the role of thermal plumes*
11:15	Vassilicos	*Are turbulent interfaces fractal or spiral?*
11:40	Stoop	*Phase transitions of scaling functions derived from experimental time series*
	Is it Chaos or Noise?	
12:05	Gorman	*Using power spectra to identify low dimensional deterministic chaos in flames*
12:30	*Lunch Break*	
2:00	Poster Session B & Discussion	
3:30	*Tea Break*	
4:00	Bertram	*The collapsible-tube oscillator as a test-bed for empirical dynamical system analyses*
4:25	Maheu	*Combined apporaches of a chaotic attractor in dynamics of a laser heated interface*
4:50	van de Water	*The shape of turbulent friction*
5:15	*Brain Break*	
5:30	Pfister	*Characterization of experimental time series from Taylor-Couette flow*
5:55	Darbyshire	*Phase space reconstruction in physical experiments*
6:20	END of DAY	

Wednesday 28 August

9:00 *Announcements*

Prediction and noise II

9:10 Grassberger *A simple noise-reduction method for real data*

9:35 Kostelich *Periodic saddle orbits, noise reduction & nonlinear prediction*

10:00 *Coffee Break*

10:25 Rabinovitch *Analysis of Noisy Signals*

10:50 Dykman *Power spectra of underdamped noise-driven nonlinear systems and stochastic resonance*

Prediction and Control I

11:15 Rowlands *Extraction of dynamical equations from chaotic data*

11:40 Gouesbet *Construction of Phenomenological models from numerical scalar time series*

12:05 Taylor *Quantifying predictability for applications in signal separation*

12:30 *Lunch Break*

2:00 Poster Session C

& Discussion

3:30 *Tea Break*

A question of dimension

4:00 Tong *On non-parametric order determination and chaos*

Prediction and Control II

4:25 Grebogi *Using time series for feedback control of chaotic systems*

4:50 Thompson *Capture, dispersal and unpredictability in chaotic dynamics*

5:15 *Brain Break*

5:30 Hubler *Optimal modeling for a control of high dimensional evolving systems*

5:55 END of DAY - but films in the evening

8:00 **Films evening** *ELT 1*

Thursday 29 August

9:00	*Announcements*	
	Fundamentals	
9:10	Pike	*Singular Systems Theory*
10:00	Sauer	*Embedology*
10:25	*Coffee Break*	
10:50	Gilmore	*Topological analysis and synthesis of chaotic time series*
11:15	Muldoon	*Topology from a Time Series*
11:40	Broomhead	*Linear filters and nonlinear systems*
12:30	*End of Day*	

Free Afternoon

1:00	Depart/Stratford	*Buses leave Mathematics Institute*
11:00	Return/ Stratford	*Buses leave Stratford*

Friday 30 August

9:00	*Announcements*	
	Natural Phenomena	
9:10	Ghil	*Unstable limit cycles, global warming and climate variability*
10:00	Kimoto	*The 40-day oscillation in the extratropical atmosphere as identified by multi-channel singular spectrum analysis*
10:25	*Coffee Break*	
10:50	Read	*Chaotic regimes in rotating, stably-stratified flow*
	Model Identification	
11:15	de Groot	*Modeling and forecasting of univariate time series by parsimonious feedforward connectionist nets*
11:40	Breeden	*A learning algorithm for optimal representations of experimental data*
12:05	Badii	*Complexity & Hierarchical Modelling of chaotic signals*
12:30	*Lunch Break*	
2:00	**Symposium Ends**	
2:05	Discussion Party for Symposium Leftovers	

A	Badan-Dangon	*Chaos in rapid tidal flows*
A	Bauer	*Empirical low-order El Nino dynamics*
A	Broggi	*Characterisation of spatiotemporal chaos in lasers*
A	Ching	*Probcbility density functions of temperature diferences in Rayleigh-Benard convection*
A	Fowler	*Random sampling and the Grassberger-Procaccia algorithm*
A	Goshen	*Calculating Fractal Dimensions*
A	Holden	*Reconstruction and quantification of attractors from spatio-temporal signals*
A	Myers	*Non-linear dynamical description of flow noise*
A	Osborne	*Fractality and chaotic dynamics in turbulent flows*
A	Otaguro	*Cognition of patterns in a fully developed turbulence*
A	Papaioannou	*Evidence of [illegible] nensional chaotic attractor in weather temperature measurements in Greece*
A	Reimers	*Experimental studies of the transiton to turbulence in pipe flows*
A	van de Water	*A dedicated computer for measuring fractal dimensions*
		Unstable periodic orbits in the parametrially excited pendulum
A	Zhang	*Estimation of the persistence of strain from experimental time series recorded from cardiac muscle during ventricular fibrillation*
B	Babloyantz	*Time series analysis of electroencephalograms*
B	Brüstle	*Deterministic properties and fractal dimension of the volcano source system from volcanic tremor records*
B	Buzug	*Calculation of optimal embedding parameters for Takens' delay time coordinates with global geometric and local dynamic methods*
B	Caputo	*Local dimensions for detailed studies of continuous and discrete systems*
B	Del Rio	*Oscillations and onset of chaos and related bifucation results for a nonlinear oscillator with possible escape to infinity*
B	Dvorak	*Corex: user friendly program for estimating correlation exponent of EL*
B	Judd	*Estimat[illegible] ion with confidence*
B	Kreuzer	*On th[illegible] nation of dimension from sampled data*
B	Landa	*Disti[illegible] shing between deterministic chaos and random noise and diagnostics of chaotic systems from time series*
		Comparison of different quantitative characteristics of chaotic motion for bubble oscillations in the sound field
B	McClintock	*Observation of stochastic resonance in a bistable system with periodically modulated noise intensity*
B	Politi	*Fractal-dimension analysis of coupled maps*
B	Sabelnikov	*Wavelet analysis of fully developed turbulence data at high Reynolds number*
B	Shaw	*Using multivariate data analysis to compare time series*
B	Stein	*Effect of quasi-monochromatic noise on nonlinear systems*
B	Tamasevicius	*A technique for measuring fractal dimensions from time series in [illegible] me scale*
B	Zheleznyak	*Dimension of spatial field distributions of nonequilibrium media*

C	Albano	*Using Neural nets to look for chaos*
C	Budd	*Time series analysis applied to a discontinuous dynamical system*
C	Eilenberger	*Bifurcation diagrams of Duffing type oscillators derive from circle maps*
C	Ghilardi	*Deterministic chaos versus random noise in rainfall time series*
C	Giacomelli	*Indictors on space complexity in extended chaotic systems*
C	Glover	*Time series near a homoclinic bifurcation*
C	Kapitaniak	*Interpretation of aperiodic time series: distinction between strange chaotic and strange non chaotic attractors*
C	Lowe	*Nonlinear Time Series Forecasting, Adaptive Networks and Numerical Optimisation Strategies*
C	Mees	*Geometry from data*
C	Parlitz	*Estimation of Lyapunov exponents from chaotic time series based on approximations of the flow map by radial basis functions*
C	Pokorny	*Evaluation of time series from forced excitable systems*
C	Sprott	*Computer package for chaotic time series analysis*
C	Stocks	*Noise induced escape across fractal basin boundaries*
C	Stoop	*Evaluation of probabilistic and dynamical invariants from finite symbolic substrings - comparison between two approaches*
C	Vautard	*Singular spectrum analysis, noise filtering, and forecasting*

PART A. LECTURES

Low dimensional model for the wall region in a turbulent boundary layer: New results

Gal Berkooz, Philip Holmes and John L. Lumley

ABSTRACT: Using an optimally convergent representation, a low dimensional model is constructed (Aubry *et al*, 1988) which embodies in a streamwise-invariant form the effects of streamwise structure (Berkooz *et al*, 1990). Results of Stone (1989) show that the model is capable of mimicking the stability change due to favourable and unfavourable pressure gradients. Results of Aubry *et al*, 1990) suggest that polymer drag reduction is associated with stabilization of the secondary instabilities, as has been speculated. Results of Bloch & Marsden (1989) indicate that drag can be reduced by feedback, and that this is mathematically equivalent to polymer drag reduction. The authors showed that dynamical systems based on the Proper Orthogonal Decomposition have, on the average, the best short term tracking time (the time that a model tracks the true system accurately; essential for control) for a given number of mode (Berkooz, 1991). In recent work, the authors have shown that several assumptions made on an intuitive basis in the work of Aubry *et al* may be justified formally. Berkooz (1990) has made rigorous estimates using the proper orthogonal decomposition showing that a structured turbulent flow, such as the wall layer, has a phase space representation that remains within a thin slab centred on the most energetic modes for most of the time. Campbell & Holmes (1990) have shown several results in connection with symmetry breaking in systems with structurally stable heteroclinic cycles. This work is relevant to our models of interacting coherent structures in boundary layers with discrete spanwise symmetry, such as that caused by riblets, which are known to produce drag reduction.

Applications of the bi-orthogonal decomposition in fluid mechanical experiments

M.P. Chauve and P. Le Gal

Most of the time, experimentalists in fluid mechanical systems encounter many difficulties in the analysis of a spatio-temporal datas. In particular, the following question is often posed: how does a flow evolve towards a fully developed turbulent regime? The aim of this paper is to present a new data analysis based on the bi-orthogonal decomposition recently proposed as a diagnostic tool for the analysis a space-time signal [1] and used for the first time to describe the transition process of the boundary layer of a rotating disk [2, 3]. First we start by a description of the method and second we give its application in two flow systems: the transition of the boundary layer on a rotating disk and the coupled wakes of 16 cylinders at low Reynolds number.

The bi-orthogonal decomposition consists in a projection of the original signal u(x, t) in spatial orthogonal modes ($\varphi(x)$) called TOPOS, and temporal modes ($\psi(t)$) called CHRONOS [1]: $u(x,t) = \sum_k \alpha^k \varphi(x)\ \overline{\psi}(t)$, the overbar denoting the complex conjugate and α the roots of eigenvalues common to both integral operators: spatial or temporal two point correlation function. Here is the analysis that we have applied in the two following flows.

One of the main interests of a boundary layer on a rotating disk is the existence, for an adapted value of the flow control parameter, of three characteristics zones, laminar, unstable with formation of spiral vortices and turbulent. The bi-orthogonal decomposition applied on temporal signals of a hot film probe placed at selected radius, gives relevant information on the dynamics of such a transitional flow. The global entropy can be introduced as a good candidate to estimate the degree of order (or disorder) of the signal.

About the coupled wakes of 16 horizontal cylinders placed in a vertical tunnel of water, the challenge was to found spatial and temporal functions (or structures) that give the total story of the mechanisms of a set of non-linear oscillators for a value of the Reynolds number just above the stability threshold. The bi-orthogonal decomposition applied on a video image of the visualization of the 16 wakes obtained by dye injection shows that only less than six modes are necessary.

It follows from these two experiments that this "beautiful tools", which extracts not only spatial structures but also temporal ones, is a precious partner to build a comprehensive route to chaos.

[1 N. Aubry, R. Guyonnet, R. Lima, "Spatio-temporal analysis of complex signals: theory and applications", to be published in Journal of Statistical physics.

[2] N. Aubry, M.P. Chauve, R. Guyonnet, "Transition to turbulence on rotating flat disk", (preprint).

[3] S. Carrion, "On the robustness of the bi-orthogonal decomposition in boundary layer of a rotating disk", (preprint).

Objective search of patterns in turbulence signals

Hiroshi Sato and Toshio Otaguro

The search of patterns in turbulent flows has a history of more than 20 years. Various kinds of pattern were reported to be found but there are strong doubts if those are real patterns. Usually the search starts by conditional sampling and ends by pattern averaging. Both procedures are highly non-convincing. The conditions for sampling are quite subjective. Different conditions result in different patterns. The meaning of pattern averaging is also unclear. Unless patterns to be averaged are very close, the averaging is just nonsense.

In a random signal or field one can find any pattern if he wants to do so. We call it "theorem of hidden pictures". Therefore, even if one finds some pattern in a turbulent flow, it does not necessarily mean that the pattern is actually there. On the contrary, it may mean that the field is random enough. Turbulence field is random but the motion of fluid is controlled by continuity condition and equation of motion. So what we expect is that some patterns may exist in turbulence with higher probability than in non-restricted random field.

In the present paper we deal with one-dimensional time series. The signal is the output of a hot-wire anemometer placed in a turbulent flow. The signal is recorded in a digital tape. The output from the tape is low-passed. The cut-off frequency is determined by the scale of patterns we are interested. We seek zero crossing of the low- passed signal. Data between two adjacent zeroes are elementary patterns. Characteristics of these patterns such as height, width and centre of gravity are calculated. Patterns with highest probability are chosen. Exactly the same procedure is repeated by the use of series of random numbers. Probabilities of finding same patterns are compared. If we find a significant difference between the two, we declare that the pattern is actually in the turbulent flow.

An experiment was made in a two-dimensional wake in a wind-tunnel. The wake is laminar to start with. By introducing sound we can excite a weak periodical velocity fluctuation in the wake. As the fluctuation travels downstream it grows and due to the nonlinear interaction low-frequency velocity fluctuations are generated. AT the same time the randomization takes place. What we are interested is to see how long the low-frequency, large-scale fluctuation can survive. Our experiment and computation based on the new method revealed that the some large-scale fluctuation survives in an apparently turbulent wake. This means that turbulence keeps memories of its birth.

We hope the present method and results cast some light on the nature of dynamical systems which include turbulent fluid motion.

A simple dynamical system that mimics open-flow ...

R. Narasimha

Flow randomization in boundary-layer transition

Y.S. Kachanov and V.Y. Levchenko

Laminar-turbulent transition is a bright example of the physical problem when a fluid mechanical system develops from a strictly ordered, deterministic state to almost completely stochastic one. The study of the routes of the flow randomization is one of the very important and very complicated physical problems. The advance in the understanding of the laminar-flow-breakdown nature is impossible now without application of different nontraditional techniques for the data processing.

The boundary layer over a flat plate was excited by a vibration ribbon to generate a sinusoidal primary instability wave. Downstream development of the mean flow and disturbances were measured by means of a hot-wire anemometer. The boundary-layer breakdown is characterized by formation of a typical three-dimensional periodic flow which is gradually breaking up into the turbulence. The appearance of nonperiodic additions, observed in oscilloscope traces, is shown in Fig. 1 where a number of the signal realizations (synchronized by the reference signal) are superposed on each graph for three downstream positions. The semi-random additions have initially a form of *stochastic anitnodes* observed within the definite part of the fundamental period. They differ ver much from the well-known turbulent spots because they are not distinguishable in any separate trace and have the same typical frequencies as the primary periodic disturbance. To understand the physical mechanisms of the flow randomization it was necessary to analyse the signals of this type. An example of such a computer data processing is illustrated by Figs. 2, 3. Figure 2 shows the results of a *two-time-scale* complex Fourier analysis of the signals. The evolution with time of the amplitudes and phases of subharmonic disturbances as the clouds of the points of phase trajectories in a complex plane (A, φ) are shown in Fig. 2b. Corresponding quasi-subharmonic traces (obtained by inverse Fourier transform) are shown in Fig. 2a. Figure 3 demonstrates downstream distributions of subharmonic amplitude, subharmonic-fundamental phase detuning and aspect ration $\varepsilon = a/b$ of the clouds of trajectory points (where a, b are the dimensions of these clouds along the main axis). It is seen that the quasi-subharmonic disturbances represent the fluctuations with subharmonic frequency having a random amplitude but definite phase, synchronized with the phase of the fundamental wave. The highest aspect ratios ε correspond to the largest amplification rates of the subharmonic amplitude. The analysis of these and others data permitted to conclude that the main mechanism of the boundary-layer randomization consists in the parametric resonant amplification of random background quasi-subharmonic disturbances as the result of their interaction with the deterministic instability wave.

Estimating the number of degrees of freedom in spatially extended systems

S. Ciliberto and B. Nicolaenko

ABSTRACT: Karhunen-Loeve decomposition has been used in order to compute the number of the fundamental degrees of freedom in a complex spatiotemporal dynamics. The method has been applied on experimental and numerical data. In both cases the number of degrees of freedom found by this method is very close to the fractal dimension of the attractor or the Lyapunov dimension.

Testing for nonlinearity in time series: The method of surrogate data

Steve Eubank, Doyne Farmer, Bryan Galdrikian, Andre Longtin and James Theiler

ABSTRACT: We are developing and testing a statistical algorithm for identifying nonlinearity in time series. Our method quantifies the confidence with which a researcher can reject the null hypothesis that a given time series might have arisen from a linear stochastic equation. The presence of nonlinearity in a time series signifies a nontrivial deterministic structure to the underlying system and suggests the possibility of chaos. More importantly, the absence of evidence for nonlinearity implies that all the irregular fluctuations of the time series can be explained as linearly correlated noise. This is particularly useful for experimental data sets, where linear correlations can often fool dimension and Lyapunov exponent estimators into signalling chaos where there is none.

The approach is related to a statistical tool that is known as "bootstrapping". For a given data set, an ensemble of "surrogate" data sets (in practice ten to fifty) are created form a linear stochastic process to mimic the autocorrelation and other features of the original data set. Computation of some nonlinear parameter (for example, fractal dimension) is performed for both the original data and the surrogate data; if significantly different results are obtained with original and surrogate data, then the null hypothesis is rejected, and nonlinearity is positively identified.

In a series of numerical experiments, we compare the relative power of dimension, Lyapunov exponent, and forecasting error for identifying nonlinear structure in a time series from various model systems under a variety of conditions. We have begun to apply our methods to real data, including sunspot cycles, river and sea levels, flame flicker, measles and chickenpox epidemics, and electroencephalograms.

Identification and prediction of chaotic dynamical systems

Leonard A. Smith

Recent advances in nonlinear dynamical systems theory have led to a multitude of calculations intended to detect low dimensional chaos in time series data of both numerical and physical origin. This paper investigates methods to quantify the significance of such calculations. Of particular interest are the analysis' of "real world" data sets, as from astrophysics or geophysics, where the parameters of the data set such as length, noise level, and sampling time are not easily varied. The basic approach is to compare reconstructions of the observed signal with those of surrogate signals which have similar temporal correlations by **no** underlying determinism. The surrogate signals are generated by Fourier transform techniques and autoregressive moving average (ARMA) models based on the observed data. One may then estimate the probability of obtaining the observed result from an ensemble of nondeterministic series. In many cases this probability is high, indicating that the results do not provide evidence of low dimensional behaviour. While the application of this approach to establishing the significance of traditional exponents (e.g., the correlation dimension) will be shown, I concentrate on applications to the analysis of dynamic reconstructions which are of use in making short term predictions. In particular, I will consider a radial basis function predictor, generalized to account for noisy data in a least squares sense, and it s response to noisy chaotic time series and their nondeterministic surrogates.

Deterministic chaos versus random noise: Finite correlation dimension and converging K_2 entropy for coloured noises with power-law spectra

A. Provenzale and A. R. Osborne

In recent works(1,2,3) we have shown that simple "coloured" random noises characterized by power-law power spectra generate a finite and predictable value of the correlation dimension and a converging value of the K_2 entropy. These noises may be expressed by the standard Fourier series

$$x(t_i) = \sum_{n=1}^{N/2} A_n \cos(\omega_n t_i + \varphi_n); \quad i = 1,\ldots,N$$

where $A_n \propto \omega_n^{-\alpha/2}$ and $\{\varphi_n\}$ are random, uniform distributed phases. By applying the well-known time-embedding method to the signal $x(t_i)$ one finds a finite and predictable value of the correlation dimension and a converging value of the K_2 entropy. These results are found as well in the case of embeddings obtained by considering several independent realizations of the stochastic process. The value of the correlation dimension is generated by the fractal (non-differentiable) nature of the signal $x(t_i)$ and it is completely determined by the value of the spectral exponent α. The convergent behaviour of the K_2 entropy is in turn associated with the convergence to zero of the "Liapunov" exponents for this class of stochastic processes. These results represent a counter-example to the traditional expectation that stochastic processes lead to a non convergence of the correlation dimension and of the K_2 entropy in computed or measured time series. These results indicate as well that the observation of a finite dimension and of an apparently converging K_2 entropy from the analysis of one or a few time series is not sufficient to infer the presence of low dimensional chaos in the system dynamics. We finally discuss the usefulness of substituting random phases to the original Fourier phases of an experimental or numerical signal in order to determine whether the fractal properties are generated by the form of the spectrum, independent of the phases (being thus associated with random noise) or if they are strictly associated with the peculiar phase relationships which are destroyed by the phase randomization (and are thus presumably due to deterministic chaos).

References

(1) A.R. Osborne and A. Provenzale, Finite Correlation Dimension for Stochastic Systems with Power-Law Spectra, *Physica* **35D**, 357-381 (1989)

(2) A. Provenzale and A.R. Osborne, Deterministic Chaos versus Random Noise: Finite Correlation Dimension for Coloured Noises with Power-Law Power Spectra, in *Dynamics and Stochastic Processes*, R. Lima, L. Streit and R. Viilela Mendes Eds., Springer, 1990.

(3) A. Provenzale, A.R. Osborne and R. Soj, Convergence of the K_2 Entropy for Random Noises with Power Law Spectra, *Physica* **47D**, 361-372 (1991).

Symmetric attractors and chaotic time series

Ian Stewart

ABSTRACT: An important technique in the experimental analysis of dynamical systems is the recognition of deterministic dynamics - especially chaos - in time series. In a dynamical system with symmetry, chaotic attractors may themselves possess a degree of symmetry. We describe some experimental situations in which this type of behaviour appears to be implicated, including patterned turbulence in fluids and nonlinear oscillations in electronic circuits. We also describe sample results from various numerical experiments.

The main point of the talk is the problem of *Equivariant Phase Space Reconstruction*. Here not only must the presence of deterministic dynamics be extracted from a time series but the symmetry of the corresponding attractor must also be obtained.

The simplest method for deducing the presence of chaos from time series goes back to Packard and Takens. It replaces the time series by a 'moving window' of vectors of fixed length N. For large enough N, generically the attractor formed by these vectors is topologically equivalent to the attractor defined by the underlying dynamics. A refinement of this method, due to Broomhead and King, applies principal component analysis to write the vectors as linear combinations of eigenvectors of a correlation matrix. Intuitively, this process finds the most common patterns among the vectors and expresses each of them as a linear combination of those patterns.

Both of these methods extend to the equivariant case. However, in order to preserve the symmetry of the attractor, we must make an *equivariant* observation - one that preserves symmetry - and *process* it equivariantly. Numerical experiments show that reconstruction methods based upon this idea do preserve the symmetry of the attractor as well as its qualitative form.

Detecting symmetry breaking intermittent chaos in turbulence

Basil Nicolaenko, Eric Kostelich and Zhen-Su She

Turbulent extended fluid systems are characterized by the coexistence of developed small-scale turbulence and intermittencies where lower dimensional large-scale structures prevail. Their transition to turbulence is often ruled by strong vorticity bursts that generate substantial spatial disorder and drive turbulence. The bursts generate a high degree of stochasticity and a large amount of enstrophy, while the flow remains dominated by large scale coherent eddies in between the bursts. Recently, within the context of periodically forced two-dimensional (Kolmogorov) and three-dimensional (Arnold-Beltrami-Childress) shear flows, we have established that such intermittent dynamics are deeply connected with the groups of symmetries of the flow. Specifically, the turbulent bursts are associated to special symmetry-breaking homoclinic cycles; the latter connect symmetry-equivariant hyperbolic states, corresponding to large scale vortices. The intermittent systems of coherent structures are not strictly identical, rather they are equivalent under isotropy subgroups action on the Navier-Stokes flow.

From the experimental perspective, several specific fluid systems are probably vested with such symmetry-breaking intermittent homoclinic chaos: transitions in Boundary Layers, unstable Tollmien-Schlichting waves, Taylor-Couette turbulence arising from unstable Wavy Taylor Vortices.

The key issue is to develop diagnostic tools to detect such symmetry-breaking homoclinicity within experimental (or computational) time-series; such series are usually affected by strong stochastic noise. Also, the system is seldom close to the critical bifurcation, the turbulent regimes prevail, while large-scale self-organization itself becomes intermittent. We present specific algorithms for the diagnostics of the above symmetry-breaking dynamics.

Modified proper orthogonal decomposition algorithms ("Karhunen-Loeve" expansions) are developed. Turbulent high-dimensional bursting regimes are filtered out, while retaining intermittent regimes with large-scale structures. The algorithms focus on the unravelling the lower dimensionality of such structures. Specifically, the algorithms must detect the subgroups of symmetry under which the large scale structures are equivariant, yet not identical from one intermittency to the next. Second, we develop noise-reduction methods to find a local linear approximation of the dynamics of the low-dimensional hyperbolic objects (corresponding to the coherent intermittent structures) and then use the above to produce new time series whose dynamics are more consistent with those near the low-dimensional hyperbolic metastable states. Both noise-reduction and proper orthogonal decomposition algorithms are combined to compute filtered dominant structures within the intermittent setting. Applications with computer movies will be presented, for two- and three-dimensional flows.

Wavelet analysis of fractal signals

A. Arneodo

In the past decade, much effort has been devoted to the characterization of fractal objects. In the context of dynamical systems theory, the Renyi dimensions have been proposed to describe the geometric and probabilistic features of strange attractors. These generalized fractal dimensions are closely related to the $f(\alpha)$-spectrum of singularities of the corresponding invariant measures. But these quantities are, in effect, statistical averages that do not provide a full description of the scaling properties of fractal objects. Our purpose is to present the wavelet transform as a multi-resolution technique which provides additional information about the spatial locations of the singularities.

First we introduce the wavelet transform as a "mathematical microscope" which is well suited to reveal the construction rules of fractals and to resolve scaling properties through the determination of local pointwise dimensions. We apply this mathematical technique recently introduced in signal analysis, to study the invariant measures of some well-known one-dimensional systems modelling the transition to chaos in dissipative systems. We emphasize the ability of the wavelet transform to reveal the renormalization operation which is essential to the theoretical understanding of the universal properties of non-equilibrium phase transitions.

We further use the extrema representation of the wavelet transform to generalize to fractal functions, the multifractal formalism originally developed for fractal measures. We describe an algorithm which can be used to extract the generalized fractal dimensions D_q and the $f(\alpha)$-spectrum of singularities directly from a velocity field of wind-tunnel turbulence at very high Reynolds numbers, without recourse to dissipation-type quantities. We compare our results to previous measurements based on dissipation type quantities.

To conclude, we present recent optical diffraction measurements of the $D_{(q)}$ and $f(\alpha)$-spectra of fractal aggregates carried out with the optical arrangement that performs the optical wavelet transform. These results illustrate the generality of our approach which is likely to extend to higher-dimensional fractal signals.

Continuous wavelet transform

A. Grossmann

Transitions in convective turbulence: The role of thermal plumes

Itamar Procaccia, Emily S.C. Ching, Petre Constantin, Leo P. Kadanoff, Albert Libchaber and Xiao-Zhong Wu

ABSTRACT: Experiments in convective turbulence have revealed a number of unexpected changes in measured signals. Most notable are (i) the transition from "soft" to "hard" turbulence at Rayleigh number (Ra) of about 10^7 - 10^8, which is seen as a qualitative change in the statistics of temperature fluctuations; (ii) a transition at Ra of about 10^{11}, seen in the Fourier spectra of temperature fluctuations measured at the centre of the medium and (iii) a change, at Ra of about 10^{13}, in the power spectra of a probe placed 0.2 cm form the bottom boundary. In this paper we present experimental evidence for the last two transitions, and offer a unified mechanism for these transitions. The main ingredient of our theoretical analysis is a calculation that suggests that isothermal surfaces wrinkle, or appear fractal, above an inner scale λ^*. This inner scale and the Hausdorff dimension of the isothermal surfaces are estimated theoretically. This scale diminishes upon increasing Rayleigh number. We argue that as it diminishes it goes through the relevant scales of this experiment, i.e. the size of the box, the mixing layer, and the height of the bottom probe. Our suggestion is that each such crossing is marked by some qualitative change in the properties of the measured signal. Thus, all these transitions may be but different manifestations of the very same physics.

Are turbulent interfaces fractal or spiral?

J.C. Vassilicos and J.C.R. Hunt

Interfaces in various turbulent flows are found to have a non-integer box-counting dimension D_K (or Kolmogorov capacity) (e.g. Sreenivasan and Meneveau 1986). They are also predicted theoretically (Batchelor 1959) to have a non-integer self-similar power spectrum; i.e. the spectrum of the scalar function F defined to be equal to 1 behind and 0 in front of the interface has, within a range of high wavenumbers, a self-similar power spectrum $\Gamma(k) \sim k^{-p}$ where p is a non-integer. In fact, above the Kolmogorov scale, D_K is found to be 2.33 (Sreenivasan et al 1989) and $p = 5/3$ (Batchelor 1959). Below the Kolmogorov scale (in the limit of infinite Prandtl number) D_K is found to be 3.00 (Sreenivasan et al 1989) and $p = 1$ (Batchelor 1959). It has also been reported (e.g. Sreenivasan and Meneveau 1986) that $D_K = D'_K + 2$ where D'_K is the capacity of the set of point intersections of the interface with a straight line.

It can be shown that all the above observations are consistent with either spiral or fractalinterfacialstructures.

(1) A spiral structure can have non-integer capacity D_K even though its Hausdorff dimensions D_H is an integer. One expects $D_H = D_K$ for fractal structures.

(2) Both spiral and fractal interfaces can be shown to have self-similar power spectra $\Gamma(k) \sim k^{-p}$ at large wavenumbers k, and p is related to the capacity $D;'_K$ by $p + D'_K = 2$. This relation agrees with the previously mentioned values taken by p and D'_K in turbulent flows; $p = 5/3$ and $D'_K = 0.33$ above the Kolmogorov scale; $p = 1$ and $K'_K = 1.00$ below the Kolmogorov scale. (In the case of Gaussian random *continuous* functions, $p + 2D'_H = 1$ where D'_H is the Hausdorff dimension of level crossings (Orey 1970, Mandelbrot 1982); this is quite different from $p + D'_K = 2$).

(3) For fractal interfaces a theorem exists which essentially states that $D_H = D'_H + 2$ (see Falconer 1985). No such theorem exists for capacities D_K and D'_K in general. Nevertheless, it can be shown that for spirals with a small number of turns (typically less than 5 or 6) $D_K = D'_K + 2$. This does not hold true for spirals with a large number of turns, in the range of very fine scales of the spiral (beyond the 5th or 6th turn).

Finally, it is shown that the range of length scales over which the exponent of the power spectrum of a spiral interface with a finite number of turns can be calculated accurately is significantly larger than the range of length scales over which D_K can be measured accurately. The value of D_K may therefore be a more sensitive measure than the value of p in indicating that some aspects of the structure of the turbulence are close to their asymptotic form for high Reynolds number.

(1) can be generalized for functions $u(x)$ that are discontinuous across spiral or fractal interfaces, but do not take only one or two values, 1 or 0, in between crossings. If we assume that the magnitude of the constant value of $u(x)$ between crossings is proportional to l^s, where l is the length between two sudden jumps of u, then u has a self-similar power spectrum $E(k) \sim k^{-q}$ with $q + D'_K = 2(1 + s)$. For a spiral interface, q can be measured accurately over a range of length scales that can be comparable to the range of length scales over which D_K is accurately measured only if $s > 0$. If $s < 0$ the error on the spectrum $E(k)$ is even larger than on the scalar spectrum $\Gamma(k)$. These properties could be used to explain recent experimental findings by Sreenivasan et al (1989) and Sreenivasan (1991) where it is reported that spectra of passive scalars can only be found to be self-similar at Reynolds numbers much higher than those where Energy spectra are found to be self-similar, and where box-dimensions D_K are equal to their asymptotic value 2.33.

Phase transitions of scaling fur.ctions derived from experimental time series

R. Stoop and J. Parisi

The occurrence of phase transitions in dyna -ical systems is intimately related to the analyticity properties of the thermodynamical functions. Such effects are believed to be of generic nature (for example, homoclinic tangency points). For one-dimensional maps, their existence has been proven for both the probabilistic and the dynamical scaling functions (that is, the $f(\alpha)$ and $\Phi(\lambda)$ spectra, respectively). With the help of simple model systems, we elucidate the different combinations in which the two types of phase transitions (namely, in the $f(\alpha)$ and $\Phi(\lambda)$ spectra) can appear. Moreover, we demonstrate the numerical evidence of phase-transition-like behaviour in both spectra derived from experimental time series of laser and semiconductor systems. Finally, we discuss new aspects for the evaluation of the thermodynamical functions from time series.

Using power spectra to identify low dimensional deterministic chaos in flames

M. Gorman and M. el-Hamdi

Our experiments on the dynamics of flat, circular premixed flames on a porous plug burner have revealed a wide variety of periodic and chaotic modes of propagation in both pulsating flames and cellular flames. In pulsating flames the flat circular flame front can exhibit these periodic modes: pulsations along its axis (axial), contractions of its radius, vibrations like a circular drumhead. It can also become 1 or 3 hot spots which rotate, form into one or more spiral arms which rotate or trace out a spiral-like path. In cellular flames the flame front physically flutes forming (bright) cells whose boundaries are demarked by (dark) cusps and folds. These cells form ordered patterns which become disordered at large values of the driving parameter. BOTH the ordered and the disordered patterns exhibit low-dimensional deterministic chaos.

With an extremely large parameter space to investigate we needed a technique for identifying deterministic chaos in the real-time environment of the laboratory. We have found that the analysis of the high frequency falloff of the power spectrum can be used to distinguish low dimensional deterministic dynamics. Sigeti and Horsthemke[1] argued that the high frequency falloff of the power spectrum should be a power-law for stochastic processes and an exponential for deterministic processes. We will present a detailed prescription for implementing this technique to analyze experimental time series. We will compare log-log and semi-log plots of power spectra, we will discuss how the high and low frequency cutoffs are picked and we will present fitting procedures which allow quantitative evaluation of the fits. We will illustrate this technique by applying it to the identification of the many chaotic states we have observed in flames. We will also present videotape of both the periodic and chaotic modes.

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ne collapsible-tube oscillator as a test-bed for empirical dynamical system analyses

C. D. Bertram

Experimental investigations of the nonlinear dynamics of fluid systems have tended to concentrate on the Taylor-Couette system and derivatives, to the point where this now occupies a position somewhat out of proportion to its importance in fluid dynamics in general. A self-excited oscillator of a different hue is observed when a fluid is forced through a flexible tube subject to sufficient external pressure to cause flattening to a non-circular cross-section. The relation between transmural pressure and cross-sectional area is single-valued but sigmoidal, with maximal compliance between first buckling and first opposite-wall contact. In this region the flow itself strongly influences the shape of the conduit. Careful experiments reveal a complex control space, with regions of divergent and of oscillatory instability. The multiple oscillatory regions suggest both different modes of a given instability mechanism and the co-exi· ·nce of more than one mechanism. A distributed rather than lumped approach to theoretical modelling is essential for the inclusion of wave-travel aspects known to be important, implying in turn a high dimension for the system. The most recent models fail to predict the observed oscillation frequencies; it is therefore not possible to regard the dynamics of such models as necessarily indicative of those of the real thing. Characterizing the dynamics of the system is further complicated by the fact that oscillations have not been observed below a Reynolds number of 500, a ıd in the present experiment, the only one thus far which documents the aperiodic oscillatory behaviour, below 5000. Thus the system is subject to a continuous random turbulent forcing. An important question is then whether the apparently chaotic dynamics can be separated form this extrinsic source. Some results of applying dynamical system analytical methods to the recorded time series have recently been published [1]; more recent investigations will be reviewed.

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Combined approaches of a chaotic attractor in dynamics of a laser heated interface

S. Meunier-Guttin-Cluzel, B. Maheu and G. Gouesbet

Unsteady behaviour of the surface of a liquid can be obtained by heating the liquid just below the surface. Heating with a focused laser beam produces either constant deformation of the surface, or periodic oscillations, or quasi-periodic oscillation or even chaotic movements depending on the laser power and the distance from the surface. The dynamical system is composed of the interface (surface tension, Marangoni effect) and of the liquid (buoyancy, thermal diffusion). The thermal gradient inside the liquid induces a refractive index gradient which produces a very nice and easily observable thermal lens pattern [1-2].

The dynamics of the heated interface is studied by means of the intensity of the laser light in the thermal lens pattern which closely reproduces the time evolution of the (hydro) mechanical system. Time series containing about 1600 000 data points have been recorded.

Beside a theoretical approach aiming at describing the experimental system by differential equations [3], we developed different and complementary approaches of the experimental data. These approaches combine spectral analysis, reconstruction techniques, (fixed radius) dimensions and entropies, recurrent plots and recurrence diagrams, periodic orbits and Poincaré sections [4]. Special attention has been paid to graphical display of the results. Hence we present a typical example of available techniques to study an original experimental dynamical system.

The results of this work show that combining different approaches of a given experimental time series allows a good insight on the structure of the underlying attractor. A single technique does not provide such a bird's eye view of the attractor although it can measure some of its characteristics. For non-specialists, combined approaches certainly lead to quicker and wider knowledge.

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The shape of turbulent friction

Willem Van de Water

We analyze multipoint measurements of the shear stress at the wall of a turbulent boundary layer. The 24 detectors ar arranged in a line perpendicular to the flow direction. The time- dependent signal is a 3D landscape that we analyze through the statistics of its iso- shear contours. These statistics have a few remarkable properties and are intimately tied to the dynamics of coherent structures in the boundary layer.

Characterization of Experimental Time Series from Taylor-Couette Flow

G. Pfister, Th. Buzug and N. Enge

Rotational Taylor-Couette flow shows a rich variety of well defined and controllable scenarios. Even for very small geometries, where the number of possible flow states is expected to be very small, period doubling cascades, intermittencies, homoclinic orbits, n-tori and mode-interaction can be found as a function of boundary conditions. In the experimental situation information about these flow states will be obtained by Laser-Doppler-Anemometry, in most cases only a single velocity component could be measured. So we are concerned with experimental noisy data points with which we have to construct a phase space. It will be discussed how electronic filtering during the experimental data recording will influence the dynamic parameters and how some of the difficulties can be overcome by filter procedures in a proper reconstructed phase space. It turned out that the optimal procedure depends on the special data set and so these will be demonstrated for various attractors.

Phase space reconstruction in physical experiments

A.G. Darbyshire and T. Mullin

In this paper we describe our experience of using phase space reconstruction methods in a variety of experimental situations. Our main motivation for performing the experiments is to relate the dynamics observed in the physical system to its equations of motion, and thus use the experiment to gain new theoretical insights. In this regard the use of time series analysis methods based on finite dimensional dynamical systems seems to provide a starting point for facilitating comparisons between experiment and theory.

To date we have considered an electronic oscillator, a parametrically excited pendulum and a fluid system, Taylor-Couette flow. One common feature of all these systems is that there is strong evidence a priori that the dynamics observed will be low dimensional. Both the oscillator and the pendulum are described by ordinary differential equations unlike the fluid system which is governed by partial differential equations, and is hence potentially infinite dimensional. However, both experiments and theoretical work [1] have uncovered features such as the interaction of symmetry-breaking and Hopf bifurcations which one might expect to be associated with finite dimensional dynamics. The application of time series analysis methods to these systems provides the opportunity to both rigorously test the suitability of a given technique and also to demonstrate its value in understanding the physical problem. An example of this latter point is the discovery of the homoclinic behaviour which organizes the dynamics in the Taylor-Couete experiment [2].

We have found that the most suitable technique for phase space reconstruction is that which uses singular value decomposition [3]. In all physical systems noise will inevitably be present. In a low noise system such as the oscillator it is possible to use linear modelling techniques to calculate the flows around fixed points in the reconstructed phase space [4]. Thus far it is not possible in the fluid experiment because of the much higher noise level.

In order to investigate the effect of noise on quantitative measures we consider the calculation of the Lyapunov exponents of the system. The method used follows that of Sano and Sawada [5] but the linearized flow map is calculated using singular value decomposition [6]. Preliminary results suggest that this method is capable of extracting the exponents from data with a significant noise content.

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A simple noise-reduction method for real data

Thomas Schreiber and Peter Grassberger

ABSTRACT: A method to reduce noise in experimental data with nonlinear time evolution is presented. A locally linear model is used to obtain a trajectory consistent with the dynamics as well as with the measured data. In contrast to previous methods where the fit to the dynamics and the cleaning are done in separate steps, it is an (iterated) one step procedure. This is made possible by using data both from the past and from the future in the locally linear model. The method is applied to both artificial and real data. Among others, it leads to a significant improvement of correlation dimensionestimates.

Periodic saddle orbits, noise reduction, and nonlinear prediction

Eric Kostelich

A new method is described for prediction and noise reduction in experimental time series whose underlying dynamics are low dimensional (chaotic or non-chaotic). The procedure uses singular value decompositions and errors-in-variables statistical methods to find linear approximations of the dynamics in neighbourhoods of the periodic orbits in an attractor. This approach is robust, computationally efficient, and generally better at reducing noise than the method described in Kostelich and Yorke (1988). Moreover, it circumvents difficulties inherent in the approach by Farmer and Sidorowich (1991), which requires accurate estimates of the Lyapunov basis associated with the tangent map at each point on the attractor.

The author will also describe how this work can be applied to the prediction of time series. In typical cases, the accuracy of a forecast depends on one's starting point in the time series. The talk will describe a way to quantify this. Examples will be taken primarily from experimental data, including chaotic chemical reactions, diode oscillators, and weakly turbulent fluid flows. Related computer software will also be described.

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Analysis of Noisy Signals

A. Rabinovitch and R. Thieberger

Autoregressive moving average (ARMA) models have been extensively used for predicting the time evolution of signals [1]. Recently the ARMA method has been applied to the analysis of chaotic data [2,3].

When a Box-Jenkins[1] or similar analysis is carried out, a specific ARMA filter is obtained as being responsible for "creating" the signal from a random shock generator. Usually there appears an added white noise as a result of measuring instruments or as an intrinsic property of the process measured itself. The amount of noise included in the signal is generally unknown, this noise brings about alterations in the ARMA parametric obtained[4] and hence also in the predicted time behaviour. Since the exact magnitude of such noise is not usually available, it is evident that a complete unravelling of the "pure" signal from the noisy one is impossible. It would seem however of advantage to have even a partial possibility of "purifying" the signal, such as a knowledge of all possible pure signals which could have played the role of our signal's origin under the addition of different amounts of white noise.

In this work[5] we present a method whereby a "partial" noise purification is performed. For the ARMA(2,2) case a complete analysis is carried out and by a nonlinear transformation the loci of the parameters for all possible pure signals, are shown to lie on a straight line. The Fourier transform and predicted time evolution can be calculated. This analysis can answer, for instance, the question whether the original signal could have been of a pure AR(2) or ARMA(1,2) type.

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Power spectra of underdamped noise-driven nonlinear systems and stochastic resonance

M. I. Dykman and P. V. E. McClintock

A survey of theoretical and experimental results on the spectral density of fluctuations (SDF) in underdamped noise-driven vibrational systems, with or without simultaneous periodic forcing, is presented. For such systems, SDFs are shown to have peculiar features that enable the interplay of nonlinearity, fluctuations and relaxation to be investigated, and which reveal some fundamental characteristics of the systems themselves. These features are also potentially of utility for practical purposes.

Depending on the noise intensity D and the character of the system there can occur several distinct peaks in the SDF. One of them is placed for small D at the eigenfrequency of the small-amplitude vibrations about the stable state. Its shape varies with D extremely strongly. In the limit $D \to 0$ it is Lorentizian, with a halfwidth equal to the damping parameter but, even for relatively small D, it has already become much broader and strongly asymmetric. The theoretical description of this peak for systems driven by a broad-band noise comes down to the solution of a boundary-value problem for an ordinary differential equation. Such solutions were obtained, analytically and numerically, for several model systems. The SDFs for these systems were also investigated in analogue electronic experiments. Theory and experiment are shown to be in extremely good agreement. It turned out, in particular, that, astonishingly, the width of the peak can sometimes *decrease* with increasing D, by a factor of two or more.

For nonlinear systems, with increasing D, in addition to the peak in the range of the eigenfrequency, there also arise peaks in the SDF near the overtones, and of particular interest, a peak at zero frequency. In contrast to the others, the latter is not broadened dramatically with increasingly D; its width remains of he order of the damping parameter.

Significantly narrower is an extra peak, also arising at zero frequency, in the SDFs of bistable systems. Its width is of the order of the probabilities of the noise-induced transitions between the states, while its intensity depends extremely (exponentially) sharply on the distance (in parameter space) from the point where the populations of the stable states are equal to each other. In periodically driven systems with coexisting periodic attractors a similar super-narrow peak is found to arise at the driving-field frequency.

A phenomenon directly related to the onset of the latter peaks is stochastic resonance (SR), the dome-shaped dependence on D of the response of a bistable system to a trial field. SR is shown to arise not only for very small frequencies, but also for those close to the frequency of a strong driving field giving rise to coexisting periodic attractors (high-frequency SR). For very weak trial fields SR is shown to be described completely by standard linear-response theory, but significant nonlinearity is demonstrated to arise for stronger (but still relatively weak) fields.

Extraction of dynamical equations from chaotic data

G. Rowlands and J.C. Sprott

ABSTRACT: A method is described for extracting from a chaotic time series a system of equations whose solution reproduces the topology of the original data even when contaminated with noise. The equations facilitate calculation of fractal dimension, Lyapunov exponents and short-term predictions. The method is applied to data derived from a numerical solution of the Lorenz equations.

Construction of phenomenological models from numerical scalar time series

G. Gouesbet

ABSTRACT: It is now well established that invariants characterizing underlying attractors may be evaluated form numerical scalar time series. The above evaluations are purely numerical and, when the algorithms are successful, there is little doubt that they provide us with valuable information. For instance, we may have definitely concluded that the system is deterministic, and also evaluated the effective number of degrees of freedom required to describe the dynamics telling us how many ordinary differential equations we need to produce a phenomenological model of the system. However, even then, we must admit that the calculated invariants are of limited practical interest, a disappointing state of affair when we consider the big amount of skill involved in the underlying mathematics and associated numerical expertise. Actually, the applied scientist would be most interested if, besides the evaluation of invariants, the available numerical scalar time series would automatically allow for the construction of phenomenological models themselves, i.e. for the reconstruction of vector fields equivalent to the original one. Our paper will address this issue. Methods for reconstructing vector fields from numerical scalar time series, a classification of the equivalent vector fields and the discussion of the meaning of the equivalence, validations relying (i) qualitatively on comparisons between graphical displays of attractors and (ii) quantitatively on systematic comparisons between the generalized dimension spectra of the various equivalent vector fields obtained in the reconstruction process, are discussed for both the Rössler band and the Lorenz butterfly. Generalizations required before applying our techniques to experimental noisy systems will be discussed.

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Quantifying predictability for applications in signal separation

William W. Taylor

ABSTRACT: Traditional signal processing techniques normally apply stochastic process theory to account for the inability to predict, control, or reproduce precise results in repeated experiments. This often requires fairly restrictive assumptions (e.g. linear and Gaussian) regarding the nature of the processes generating the signal source and its contamination. Our purpose is to provide a preliminary analysis of an alternative model to account for this random behaviour. The alternative model assumes that randomness can result from chaotic dynamics in the processes which generate and contaminate the signal of interest. This provides the option to use nonlinear dynamic prediction models instead of traditional statistical modelling for signal separation. The effectiveness of a given prediction model for a particular application can then be interpreted in terms of the predictability of the data set using that model.

On non-parametric order determination and chaos

B. Cheng and H. Tong

We have given a brief introduction to deterministic chaos, highlighting the connection between some standard one-dimensional chaotic models with stochastic time series models via time reversal. We argue that it is often natural to determine the embedding dimension in a noisy environment first in any systematic study of chaos. We have introduced the notion of a generalized partial autocorrelation and an order relevant to nonlinear autoregressive modelling. We have described statistical approaches to order determination of an unknown nonlinear autoregression and have given justification including consistency and some illustration based on the Hénon map. The downward bias of the residual sum of squares as essentially a noise variance estimator is quantified.

Using time series for feedback control of chaotic systems

Celso Grebogi

A method is proposed whereby motion on a chaotic attractor can be converted to a desired attracting time-periodic motion or steady state by making only *small* time-dependent perturbations of some set of available system parameters. By using delay coordinate embedding, the method is applicable to experimental situations in which *a priori* analytical knowledge of the system dynamics is not available. Important issues include the time required to achieve control, the effect of imperfect system identification, and the effect of noise. The method is illustrated by applying it to a physical experiment.

Capture, dispersal and unpredictability in chaotic dynamics

J.M.T. Thompson

The three central concepts of nonlinear dissipative dynamics are attractors, basins and bifurcations: and chaos, with its intrinsic sensitivity to initial conditions and noise, impinges on each of these to generate different aspects of chaotic unpredictability. The chaotic attractor captures all trajectories within its basin and disperses them by a spreading, folding and mixing action within a fairly well-defined, albeit fractal, topological structure. Chaotic transients, associated with chaotic saddles in fractal basin boundaries give a similar but transitory action, before dispersing trajectories to two or more distinct and remote attractors: here a much greater dispersal is achieved, but only trajectories within the fractal boundary zone are involved. Indeterminate bifurcations, such as the recently explored tangled saddle-node bifurcation [1, 2], can be seen as a mechanism for sweeping all trajectories from a phase-control basin precisely onto a fractal basin boundary, from which they are dispersed to two or more uncorrelated and remote attractors. These bifurcations thus achieve the same high degree of dispersal as a fractal basin boundary while capturing, during a slow parameter evolution, a full basin of trajectories: they thus generate, in a very real sense, the highest degree of unpredictability in regular and chaotic dynamics. This paper explores these ideas in some detail, and introduces a capture-dispersal diagram to assess the relative degrees of unpredictability in the above attractor, basin and bifurcation scenarios.

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Optimal modelling for a control of high dimensional evolving systems

Alfred Hubler

At least for short periods of time, the dynamics of many complex systems can be estimated from a low dimensional differential equation or map, although the numbers of degrees of freedom is large. Nearly all variables are slaved by a few order parameters. However, if the complex system is perturbed by an external force for a control, slaved variables can be stimulated and a prediction of the response becomes impossible. We show that it is possible to predict the response of the complex system and to control it, if the driving forces are resonant perturbations of an appropriate low dimensional model.

We have developed a method for reconstructing those appropriate models. In contrast to previous modelling techniques this method does not rely on delayed variables. It seeks to optimize the state space representation and the modelling error simultaneously and takes full advantage of any available preknowledge about the dynamics of the experimental system. The technique can be applied to systems with one or many hidden variables and is robust against noise.

Singular Systems Theory

E. R. Pike

An account will be given of a programme of work which has been pursued in recent years by the author and his colleagues which starts from the famous solution of the first-kind Fredholm equation for the prolate spheroidal functions by Slepian and Pollak in 1961. This provides the solution to an inverse mapping problem from L2 into itself. We have extended this work in a number of directions to deal with general inverse mappings between different function spaces and used it to provide a general theory of information, applicable to diverse problems including time series analysis. We have been particularly interested, however, in light scattering spectroscopy and scanning confocal microscopy and will illustrate the lecture with results in these areas.

Embedology

Tim Sauer, James A. Yorke, Martin Casdagli

Mathematical foundations of the embedding methods commonly used for the reconstruction of attractors from data series are discussed. The basic results are generalizations of the previous embedding theorems of H. Whitney and F. Takens. The generalizations proceed in three directions.

First, the set to be embedded may be a fractal attractor instead of a manifold. In this case, if n is an integer larger than twice the box-counting dimension of the fractal set A, then almost every reconstruction map to R^n is one-to-one on A, and moreover is an embedding on smooth manifolds contained within A. Additionally, under fairly weak hypotheses, the same is true for almost every delay-coordinate map into R^n. Examples show that these statements fail if box-counting dimension is replaced by Hausdorff dimension.

Second, we study reconstructions which use moving averages of delay coordinates. Again, necessary conditions (relating to the periodic orbits of the dynamical system) on the moving average filter are given so that the delay-coordinate map is an embedding for almost every observation function. Third, in the case that n does not exceed twice the box-counting dimension of A, information on the resulting self-intersection set can be deduced. For many applications, something less than an embedding may suffice.

The use of the term *almost every* in the above statements is in the sense of *prevalence*. We say that a subset S of the infinite-dimensional space of smooth maps is prevalent if there is a finite-dimensional subspace E which intersects S in full measure, and if every finite-dimensional subspace containing E also intersects S in full measure. Prevalent subsets of reconstruction maps are in particular dense, but the probability-one notion of prevalence is much stronger, since it is possible for dense subsets to have zero probability.

A couple of related results deal with the preservation of fractal dimension under reconstruction. If A is an attractor of Hausdorff dimension d and if $n > d$, then for almost every (in the sense of prevalence) smooth reconstruction map F into R^n, $HD(F(A)) = HD(A)$.This is a generalization of an existing result, which states that the equality holds for almost every linear projection F. Surprisingly, this statement fails if Hausdorff dimension is replaced by box-counting dimension.

Topological analysis and synthesis or chaotic time series

Robert Gilmore

We have developed topological methods both to analyze and synthesize chaotic time series data. These methods have been used to analyze data form the Belousov-Zhabotinskii reaction and the Laser with Saturable Absorber. In this analysis, almost periodic orbits are extracted from the chaotic time series by a search for close returns. A differential phase space embedding in R^3 is constructed using a simple differential-integral filter. The linking and self linking numbers and the relative and self relative rotation rates of all periodic orbits and pairs of orbits are then determined. From this information the template supporting the chaotic attractor is determined and verified. The embedding and template determination can be done on line for driven systems and, with a little care, for autonomous systems.

Chaotic time series can be synthesized for flows supported by any template. It is sufficient to specify the template matrix and array to construct such flows. The template provides a geometrical model for the strange attractor. This model is useful for visualizing the 'unfolding' of the strange attractor. In this way we have determined processes resp(nsible for the creation of bifurcation bubbles, for the evolution of the Lorenz system at high r values, for the existence of 'peninsulae' in the parameter space of the Duffing oscillator where the global torsion changes by 2 systematically, and for the effects of symmetry breaking.

Software for performing the following functions will be available:

1. Search for close returns: extracting segments of chaotic time series which mimic periodic orbits.
2. Differential phase space embedding.
3. Linking numbers: measurement from pairs of extracted periodic orbits.
4. Relative Rotation Rates: measurement from pairs of extracted periodic orbits.
5. Prediction of Linking Numbers: inputs are a template matrix and list of orbits.
6. Prediction of Relative Rotation Rates: inputs are a template matrix and list of orbits.
7. Synthesis of chaotic time series: inputs are a template and values for 'pruning parameters'.

Topology from a time series

D.S. Broomhead, J.P. Huke, R.S. MacKay and M.R. Muldoon

Takens embedding theorem tells us that a time series measured from a dynamical system contains a great deal of information; the usual delay coordinates provide an embedding of the system's invariant manifold into Euclidean space. We construct these manifolds directly from time series data, producing two representations, one as the intersection of zero sets of functions and another as a triangulated hypersurface. With these we identify the manifold's topological type and construct canonical coordinates. In each case we use principal component analysis to perform the construction locally, building tangent spaces to small neighborhoods, then glue the pieces together to make a global object.

Linear filters and nonlinear systems

D.S. Broomhead, J.P. Huke and M.R. Muldoon

It has been asserted, in work by Badii and Politi and co-workers [1, 2] and Mitschke and co-workers [3, 4], that the low-pass filtering of time series data may lead to erroneous results when calculating attractor dimensions. Their work applies to recursive filters - here we prove that finite-order, non-recursive filters do not have this effect. In fact, a generic, finite-order, non-recursive filter leaves invariant all the quantities that can be estimated using embedding techniques such as the method of delays [5].

Further, we exploit the relationship between recursive filters and infinite-order, non-recursive filters to study the conditions that must be satisfied by recursive filters in order that they too preserve these quantities. In this way the connection is made with the work of references [1, 2, 3, 4]. Moreover, it is shown that the conditions given in references [1, 2, 3, 4], which ensure the preservation of dimension, are not sufficient to ensure an embedding.

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Climatic trends and variability: Diagnosis and prediction

Michael Ghil

The standard view of trends and variability has been based on the dichotomy that trends are deterministic and predictable, while variability is random and unpredictable. In fact, these two are but extremes of a spectrum of types of climatic behaviour, which extends from linear (or exponential) trends through periodic, quasi-periodic and deterministically aperiodic behaviour to the truly stochastic; all but the first of these make up the variability.

I shall introduce a method for decomposing the climatic signal into a trend, a (quasi-)periodic and aperiodic part, and a white-noise residue. The method, singular spectrum analysis (SSA), is based on the Karhunen-Loève expansion of a time series and produces data-adaptive sets of weights for running averages, and hence trend detection. It also produces data-adaptive basis functions for the oscillatory part, i.e., the nonlinear counterpart of sine-and-cosine pairs, permitting the separate reconstruction of any given part of the time series including its amplitude modulation.

SSA is applied to climatic time series on various time scales: Quaternary isotopic records from hundreds of thousands to millions of years, global temperature records over one-and-a-half centuries, and sea-level pressure records associated with the El Niño-Southern Oscillation (ENSO). For the temperature records, SSA shows clearly a rising trend of 0.5°C per century, concentrated between 1910 and 1940, and since 1980. SSA is combined with the multi-taper method to produce high-resolution spectra of these signals, yielding interdecadal oscillations with a period of 21-23 and 16 years and interannual, ENSO-related oscillations with a quasi-biennial period and a four-to-six year period. SSA is combined with the maximum-entropy method to yield more robust and reliable statistical predictors than heretofore available for the oscillatory part of the ENSO signal, predicting a minor warm event in 1991.

The 40-day oscillation in the extratropical atmosphere as identified by multi-channel singular spectrum analysis

M. Kimoto, M. Ghil and K.-C. Mo

ABSTRACT: The three-dimensional spatial structure of an oscillatory mode in the Northern Hemisphere (NH) extratropics will be describe.d The oscillation is identified by using a multi-channel version of the singular spectrum analysis (SS), which has been discussed by Broomhead and King (1986) and by Vautard and Ghil (1989). This study is a continuation and extension of previous work by Ghil and Mo (1991; GM) who used single-channel SSA to identify a 40-50 day mode in two of the leading principal components (PCs) of NH 700mb height fields. The multi-channel SSA (M-SSA) analyzes a matrix whose elements are temporally-lagged covariances between all the possible pairs of spatial variables considered. Therefore, it is suitable for detecting and describing weak oscillations in highly chaotic background.

The extratropical 40-50 day oscillation found by GM has been confirmed by M-SSA and its spectral peak has been narrowed to 40 days. It is described in this study by constructing composite maps keyed to the extratropical multi-channel PC. M-SSA has been applied to the first eleven NH 700 mb EOFs, based on 40 years of daily maps; these eleven explain 67% of total variance, when the seasonal cycle is removed and no seasonal stratification is performed. Global atmospheric circulations associated with this oscillation are studied by constructing eight-category composites of various variables, e.g., upper-air winds, temperatures, and a tropical convection index, using more than 10 years of data for each of them.

Major features of the composites are: (i) two wave-trains, one over Pacific/North America and the other over Eurasia, alternate during the cycle; (ii) weakening and strengthening of the westerly jet over the Atlantic sector; (iii) retrogression of anomaly centres from northern Europe all the way to the northeastern Pacific; (iv) two distinct cold surges in winter, one over the Mid-Pacific trough and the other over the Asian Far East; and (v) equatorward propagation of zonal mean winds and angular momentum. Some of these features are consistent with recent results by GM and other investigators. Interaction with another tropical oscillation with similar frequency is also described.

Chaotic regimes in rotating, stably-stratified flow

Peter L. Read

Thermal convection in a rotating, cylindrical fluid annulus, differentially heated in the *horizontal* ('sloping convection'), has been studied for many years[1] as a simple laboratory analogue of the large-scale circulation in a planetary atmosphere. Like the more widely familiar Taylor-Couette system, it is characteristic in its typical form by circular symmetry in its boundary conditions, and is well known to exhibit a rich variety of regimes of flow depending upon the external conditions (e.g. temperature contrast ΔT and rotation rate Ω). These include steady axisymmetric flow (analogous to Hadley flow in the Earth's tropics, at low Ω), spatially-regular and temporally steady or periodic non-axisymmetric baroclinic waves (at moderate Ω), and (at high Ω) spatially irregular, aperiodic baroclinic flows (a form of fully-developed geostrophicturbulence).

Results will be presented from analyses of high-precision time series of measurements of temperature and total heat transport obtained in a moderate Prandtl number ($Pr = 26$) fluid contained in a rotating, cylindrical annulus, concentrating on regions of parameter space close to observed transitions to chaotic behaviour. Temperature measurements provide information on the local behaviour of each flow, while the total heat transport forms a global measure. The spatial characteristics of the flows have also been studied using regular arrays of thermocouple probes. Two main regions of parameter space in which a transition from quasi-periodic to chaotic behaviour occurs have been identified[2]. The first occurs in an isolated region of parameter space at moderate/high Taylor number, apparently associated with a transition to a lower azimuthal wavenumber, in which a quasi-periodic, amplitude-modulated travelling wave (2-torus) gives way to a low-dimensional ($D \sim 3$) chaotically-modulated vacillation (chaotic 3-torus?) at very low frequency, suggestive of competition between modes with adjacent azimuthal wavenumbers. The other main transition to disordered behaviour is associated with a weak temporal modulation of azimuthal (and radial?) harmonics of the dominant wave and exhibits some characteristics suggestive of temporal intermittency close to the transition itself.

Various signal processing methods have been used to characterise the flows, including 'phase portrait' reconstruction techniques and subsequent estimates of attractor dimension and Lyapunov exponents. Emphasis is placed on the advantages of applying singular value decomposition[3] to the more widely used analysis techniques, and an assessment of the relative merits and demerits of such methods will be discussed.

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Modelling and forecasting of univariate time series by parsimonious feedforward connectionist nets

Claas de Groot and Diethelm Wuertz

ABSTRACT: We report on a parsimonious feedforward connectionist net approach for the modelling and forecasting of time series. The methodology includes all states of a nonlinear time series analysis: "model identification", "model building" and "diagnosis checking".

Within the model identification process we present tests and methods to investigate the character of patterns appearing in the time series and the character of the time series itself. Qualitative and quantitative measures for randomness, complexity, dependencies and nonlinearities are derived and discussed. From this we get first information how to design the connectionist net.

Model building and parameter estimation is the second step. We have investigated a series of different training algorithms ranging from "backpropagation", via "standard optimization" procedures (direct search methods, modified-Newton and quasi-Newton methods, least square algorithms, conjugate gradient methods) to "stochastic optimization" algorithms (simulated annealing, evolution strategies, Langevin dynamic algorithm). In the optimal case we achieve performance improvements in CPU time up to a factor of 1000 in comparison to standard backpropagation. However, training a net cannot only be reduced to the question of optimization concepts. We have investigated also other important influences coming form the special choice of a performance measure, of a net transfer function and of the starting configuration for the optimization process.

Once we have chosen a special topology for the net and estimated its weights and threshold values we have to check its adequacy. In this context we present tests and algorithms to obtain quantitative measures. These methods are based on concepts of overfitting, on the investigation of residuals, and on the use of an information criterion. Int he ollowing forecasting process the principle of parsimony becomes evident. This principle is important because, in practice, parsimonious models generally produce better results. Thus the idea of parsimony gives our modelling procedure a strong practicalorient. .on.

Within our approach we present three applications: Time series analysis and forecasting of sunspots activity data, noise reduction and signature verification. In the first case we show that the connectionist network approach performs better than traditional non-linear time series methods e.g. bilinear and threshold autoregression. In the second application we demonstrate that connectionist nets are able to distinguish "real white noise" from a deterministic process which also shows a flat spectrum and delta-correlated autocorrelations. This result may be useful in developing a new method of noise reduction suitable for problems intractable by standard methods. In the dynamic signature verification example we discuss how the benefits of the two well established methods of function comparison and parameter comparison can be combined.

A learning algorithm for optimal representations of experimental data

Joseph L. Breeden and Norman H. Packard

We have developed a procedure for finding optimal representations of experimental data. *Optimal* is defined as the state space representation that best suits one's stated goal. Conceivable goals could be to facilitate calculation of invariant measures of the system, determinism, forecasting, control, and others. This goal is translated into a functional which, when applied to trial representations, computes a quality that is used to compare their suitabilities. A learning algorithm based upon the genetic algorithm uses this quality function to search the space of possible representations (dimensions and coordinates). We include in the search a broader class of coordinates than the derivatives and delays normally used. Any function on the original data set is a candidate, such as integration windows or coordinates with redundant information. Mixed representations incorporating coordinates of different types may be advantageous in certain situations. This method is particularly useful to sparse or noisy data, because it finds the representation in which the available data best satisfies the given criterion. Numerical experiments have shown that in almost all cases, no one representation is optimal for all the goals mentioned above.

We have extended this technique to treat data which is nonuniformly sampled in time by generating fuzzy delay coordinate representations. Rather than choose a precise delay, the program searches for a delay, τ, and a window about that delay, $\delta\tau$, according to which points are accepted for the state space representation. This is an attempt to maximize the amount of usable data while minimizing the loss of information. We can address situations where interpolation to uniformly spaced data is not possible equally well.

We illustrate these techniques with a range of model examples and also to sparse, nonuniformly sampled observations of quasar B1 1308+236. Our methods give strong evidence for the existence of predictability of the quasar oscillations (the chance that the data represents random fluctuations is less than 1 in 7,000). To date, this is the first evidence that quasar oscillations may not be random.

Complexity and hierarchical modelling of chaotic signals

R. Badii and M. Finardi

We present a new method for charcterising the complexity of chaotic experimental signals in terms of the accuracy of descriptions obtained with a hierarchical modelling procedure. This is based on a symbolic encoding of the trajectories in embedding space. The identification of all unstable periodic orbits, up to a given order, allows us to approximate a generating partition on a suitable Poincaré surface.

The languages underlying the associated symbolic dynamics are unfolded by constructing prefix-free codes and described by means of "logic" trees. The conditional probabilities for sequence-to-sequence transitions are estimated by considering orbits belonging to adjacent tree-levels. Accordingly, various types of Markov models are obtained and used to predict the asymptotic limit of thermodynamic sums, within a grand-canonical formulation.

A considerable improvement is achieved by recoding the original signal in terms of variable-length words and by re-applying the above procedure to the transformed signal, which is equivalent to a renormalization operation on the associated dynamical map. Generalized dimensions and entropies are evaluated by using a transfer-matrix formalism. The accuracy of these estimates is directly related to the convergence of appropriate scaling functions, through a measure of complexity.

The procedure has been tested on time-series generated by the Lorenz and Rössler differential flows and has been applied to the chaotic output signal of an NMR-laser. The validity of modified phenomenological Maxwell-Bloch equations is demonstrated by showing that they exhibit the same topological structure as the experimental data.

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PART B. POSTERS

Chaos in rapid tidal flows

Antoine Badan-Dangon

Experimental data series of rapid ocean currents were collected at the southern end of Ballenas channel, in the Gulf of California, in a region where the flow accelerates and plunges over a sill, in the form of a hydraulic jump. The series are used to reconstruct the evolution in phase space of the flow, following techniques proposed for systems with a single observable quantity. Attractors of different sizes are isolated. The largest is periodic, non-chaotic, and represents the tidal portion of the ocean currents. The second is chaotic, with a Hausdorff-Besicovitch dimension $d \sim 2.62$, and a maximum positive Liapunov exponent $\lambda = 10$ bits per orbit. It represents, possibly, the effects of the strong turbulent mixing, which is thought to be predominantly responsible for the dissipation of the tidal energy. The results are confirmed by repeating the analysis with other observables and, finally, by taking all available series to reconstruct the phase space portrait of the attractors. All series provide descriptions which are topologically equivalent. This exercise contributes to the study of the connection between the Navier-Stokes equations and the properties of finite dimensional dynamical systems.

Empirical low-order El Nino dynamics

Sonia T. Bauer and Michael G. Brown

We seek to determine the extent to which temperature fluctuations in the equatorial Pacific Ocean, and, in particular, the occurrence of El Nino events, can be modelled as a low-order dynamical system. Vallis (Science 232, 243-245, 1986) has proposed such a low-order model in which the basin scale zonal temperature gradient is coupled to the wind-driven zonal currents via fluctuations in the trade winds. The existence of such a low-order dynamical system is of interest for several reasons. Firstly, the existence of such a system of equations means that El Nino events can be accurately modelled using a severely truncated physical model. Secondly, the existence of such a model guarantees short term predictability of El Nino events. And thirdly, if solutions to the system of equations admit chaotic solutions, as did the Vallis system, then long-term predictability is precluded.

To investigate low-order El Nino dynamics we employ an extension of the Broomhead and King (Physica 20D, 217-236, 1986) phase space reconstruction technique. This technique consists of projecting a measured time series onto a number of basis functions (temporal empirical orthogonal functions), each consisting of an eigenvector of a time-lagged time series covariance matrix. The resulting projections yield a multidimensional phase space portrait of the underlying dynamical system. The extension which is employed utilizes projections of vector time series onto spatio-temporal empirical orthogonal functions as a means of reconstructing the phase space portrait. This technique is applied using 100 year long time series of monthly averaged sea surface temperatures in the central and eastern equatorial Pacific Ocean.

Characterisation of spatiotemporal chaos in lasers

G. Broggi

Transverse instabilities in lasers can give rise to structures that are irregular both in space and in time. These phenomena can occur also when there is evidence that only relatively few cavity modes are active, thus suggesting a clue with low-dimensional deterministic chaos. The characterisation of spatiotemporal irregular structures requires the application of concepts from the theory of low-dimensional attractors (dimension analysis) and from information theory (mutual information) and brings about severe computational difficulties. The results of our analyses, especially for what concerns the relation between mode excitation and relevant active degrees of freedom, sheds new light on the behaviour of extended nonlinear systems.

Probability density functions of temperature differences in Rayleigh-Benard convection

Emily S.C. Ching

I report here the behaviour of the probability density functions (PDF) of temperature differences, between different times, but measured at the same point, which is at the centre of a helium-gas cell. One objective of this work is to study how thee PDF evolve as the separation time increases. I studied data from seven Rayleigh numbers (Ra) which range from 10^9 to 10^{15} and are above what is called the soft to hard-turbulence transition. These PDF are symmetric and non-Gaussian which are fitted approximately by a stretched-exponential form, $e^{-c|x|\beta}$. As the separation time τ increases, the parameter β starts from 0.51 ± 0.05 (for the smallest separation), remains approximately constant for $\tau \leq \tau_1$, then increases, and finally at $\tau = \tau_2$, it saturates to 1.7 ± 0.1 (1.6 ± 0.1 for the two largest Ra studied). For $Ra < 7.3 \times 10^{10}$, β increases $\tau^{0.27\pm0.03}$ while for $Ra \geq 7.3 \times 10^{10}$, the increase has to be described by two powers: for lower τ, β first increases slower as $\tau^{0.15\pm0.03}$, then for $\tau \geq \tau_b$, it increases as $\tau^{0.27\pm0.02}$.

Random sampling and the Grassberger-Procaccia algorithm

G. Kember and A.C. Fowler

The correlation integral of a data set is determined from information based upon a random ample of it. Given a data set of a certain size, we use sampling statistics to specify the sample size required to infer the correlation integral to a desired accuracy. The minimum sample size required to resolve a correlation exponent is found by consideration of both data set and sampling errors.

Calculating fractal dimensions

S. Goshen and R. Thieberger

In the past few years much attention has been paid to the study of fractal sets[1] in a metric space such as a geometric object or the phase space trajectory of a dynamical system. It was shown[2] that the Renyi dimensions D_q are good candidates to characterize the statistical distributions of these sets of points. There have been many calculations of these dimensions. In this work we use the box counting algorithm, which has gained renewed interest in recent years[3-6].

We wish to show how one can construct a program that saves memory for a large array of data. We also used transputers and perform a parallel implementation of the algorithm presented here, thus reducing computation time at a relatively low cost. The basic method used is an extension of previously discussed algorithms[5,6].

The set we are concerned with has to be embedded in a finite grid of boxes, whose dimension is the dimension of the embedding metric space. We wish to stress that for the usual fractals encountered only a small number of boxes from the total, are occupied. Therefore instead of using predefined boxes, they will be defined ad-hoc. A box is defined when some point has to be put into it.

The whole algorithm consists of the following parts:

a. All points are scanned so that their maxima and minima can be found.

b. The double precision sets of points (, Y) are mapped into the space of 31-bit non-negative integers (IY, IY). We have now to choose the maximal divisions of the box axis to be less or equal to 2^{15}, under this condition we can map the (IX, IY) to a new set of integers IXY so that two different points (X, Y), X', Y') which are mapped to the same IXY are in the same box. The next step is to build up an array defining all the occupied boxes and a second array listing the number of times an occupied box appears. To enable handling large numbers of points, one can handle subsets of the total set of points and merging the results.

c. One then calculates all the desired D_q, for different values of q, and then merge boxes so as to obtain coarser divisions.

Parallel implementation was done using EXPRESS-3L FORTRAN[7,8], the main improvement was in part b., which is the most time consuming.

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Reconstruction and quantification of attractors from spatio-temporal signals

A.V. Holden, H. Zhang, I. Dvorak and H. Pockberger

Recordings of the gross electrical activity of the brain, as clinically or experimentally obtained electroencephalograms or electrocorticograms, have traditionally been analysed as stochastic processes, using frequency domain methods. Recently there has been interest in analysing these signals using methods derived from nonlinear dynamics, and strange attractors have been reconstructed and quantified, and changes in the dimension of these strange attractors have been found to be associated with pharmacological interventions, pathological processes and attentional states [1].

These irregular recordings are usually from several sites - a standard EEG recording is on 19 channels from anatomically defined recording sites, and so reconstructions can be from a time series recorded from a single site, or from simultaneously recorded multiple time series. These recordings provide a useful testbed for evaluating these different approaches of attractor reconstructions, as the brain, and its activity, has a high degree of spatial structure, and the multi-channel recordings provide a sampling of a irregular, spatio-temporal process where there are functional dependencies between the behaviour at functionally related sites, but not between functionally disparate sites.

From the recorded multichannel time series $x_i(j\tau)$, $j = 1, 2, 3, ..., n$, τ is the sampling interval, Takens [2] provides a time delay method to reconstruct the attractor from a single time series. A point in the attractor is

$$\mathbf{x}_i(j) = (x_i(j\tau),\ x_i(j+1)\tau),\ ...,\ x_i((j + (M-1))\tau) \tag{1}$$

where M is embedding dimension. An alternative is the multichannel method proposed by Eckmann and Ruelle [3]. A point in the attractor is

$$\mathbf{x}(j) = (x_1(j\tau),\ x_2(j\tau),\ ...,\ x_M(j\tau)). \tag{2}$$

In the context of EEG studies, multichannel reconstruction and Takens reconstruction are compared.

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Non-linear dynamical descriptions of flow noise

Dr. Doug Mook and Dr. Cory Myers

Techniques in non-linear dynamics have been developed in the past several years which have enormous promise for modelling broadband stochastic processes. Unlike previous techniques which describe stochastic processes in terms of power spectra, descriptions using non-linear dynamical methods are effective for non-Gaussian processes and can often effectively act to predict and smooth data which is apparently chaotic. Such broadband processes are, in fact, deterministic, not stochastic, and this is the feature which allows progress to be made in their study.

These techniques are of value for physical processes which are of low or moderate dimensionality. Low dimensional dynamical processes frequently are often described theoretically by very high order differential equations, but have asymptotic behaviour which is locally of low or moderate dimension in phase space. This contraction of the full state space to an *attractor* comes from the dissipative, driven aspect of the systems which is mirrored in practice.

We have been working with wind noise as an example of such a process. Recent work indicates that wind noise can be modelled as a low dimensional non-linear dynamical process [1]. We have made high quality measurements of wind noise and various approaches were used to estimate the dimension, the degree of correlation, and the mutual information. Estimation of the dimensionality of wind noise is complicated by the fact that additive high dimensional noise is also present and even at low levels makes many common methods for estimating dimensionality unworkable. The correlation provides information about the degree to which linear models (or time-varying linear models) are capable of effectively modelling the data. The effectiveness of these linear models, which are optimal for the case of Gaussian processes, provides a baseline against which we have been benchmarking the non-linear dynamical modelling techniques. Finally, the mutual information provides an effective measure of predictability (or model effectiveness) which indicates fundamentally how well the process can be described and how close the non-linear dynamical model is coming to that fundamental limit.

We have found that under many conditions and appropriate measurement techniques, **wind noise is a low dimensional chaotic process**. We have developed a novel method of using a classifier to build a non-linear dynamical model of the process. Mutual information, estimated during the training of the classifier on the data, is used to determine the dimensionality of the process. The non-linear dynamical model developed by the classifier for the process can also be used for prediction and smoothing. Following techniques described by Marteau and Abarbanel [2], we are using this model to separate the dynamical process from additive signals.

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Fractality and chaotic dynamics in turbulent flows

A.R. Osborne

Recently a large number of experiments have been conducted for which finite values of the correlation dimension ν and the K_2 entropy have been found, and for which the discovery of a strange attractor has been claimed. A subset of these measurements have power law spectra, $f^{-\alpha}$ (α = const), and uniformly distributed random Fourier phases. Unfortunately these latter systems are *not necessarily describable by a strange attractor* [1,2] and instead have been classified as *more stochastic in nature than dynamic*. Typically, measured signals of this kind are also *fractal functions of time or space* [3].

The disconcerting discovery of *flows with a fractal dimension without a strange attractor* emphasizes the need for developing certain limited properties of a nonlinear system, but we recognize that *the flows must be essentially dynamic* and therefore that a dynamical description is mandatory for understanding their behaviour. Here we present new time series analysis methods in order to assess the underlying dynamics of measured nonlinear flows of this type.

In order to address directly the dynamics of "stochastic" flows, and their anamalous fractal behaviour in the absence of a strange attractor, we present a prototypical Hamiltonian dynamical system which exhibits most of the properties of the experimentally investigated dynamical systems discussed above [3, 4]. This system is given by the following *Hamiltonian* equations of motion:

$$\dot{\mathbf{x}} = u_{rms}\hat{\mathbf{k}} \times \nabla\psi(\mathbf{x}, t) \tag{1}$$

The model predicts nonlinear fluid parcel trajectories in 2-D fluid turbulence where the stream function power spectrum has the form $k^{-\gamma}$ (γ const). The over-dot denotes time derivative, $\mathbf{x}(t) = [x(t), y(t)]$ is the parcel position, $\hat{\mathbf{k}}$ is the unit vector perpendicular to the xy-plane, $\nabla = (\partial_x, \partial_y, \partial_z)$ and u_{rms} is the rms velocity. $\psi(\mathbf{x}, t)$ is the stream function or Hamiltonian of the flow. The dispersion relation is $\omega = uk$, where $k = |\mathbf{k}|$, u is constant. Using this model we apply both analytical and numerical methods to show that fluid particle trajectories: (1) while differentiable and chaotic at small scale, may be *fractal space curves* at larger scale and (2) undergo *anamalous transport*.

Flows of this type are *among the most common known fluid dynamical motions* (think of any "turbulent" system where power-law spectra dominate, such as in 2 or 3D turbulence, climate or weather dynamics, internal waves, geophysical fluid dynamics, etc.). For this reason we have developed *new dynamical time series analysis methods* which enable the investigator to analyze turbulent-like fluid systems

[3, 4]. The methods, generalisations of the Grassberger and Procaccia approach, may be interpreted not only in terms of *dynamical systems properties* such as ν, K_2, (multivariate scaling) but also in terms of *anomalous diffusion* in the flow (time-embedded, time-sliced Poincaré diagrams) [3, 4]. Ambiguities in the interpretation of "flows with a fractal dimensions without a strange attractor" are largely removed.

We would enthusiastically participate in the proposed program to analyze data with our new approaches.

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Cognition of patterns in a fully developed turbulence

Toshio Otaguro

Is objective search of coherent patterns possible in turbulence signals? This is one of the questions we have been asking in various situations of data reduction of turbulence. We had found an empirical law called 'Theorem of Hidden Pictures,' which states that in a sufficiently turbulent signal you can find any pattern you want. The theory taught us the difference between *cognition* and *recognition*. Cognition is to define categories, and recognition is to sort data into categories. The former is quite a human and intuitive process. It poses a serious question on the validity of the conditional sampling technique. For example, you can even set a condition of your own type without physical significance, and perform sampling and averaging. Then you will get a pattern, which, however, is almost useless. The problem can be boiled down to the question whether you can find a significant pattern out of a random field without cognition or *a priori* knowledge.

Averaging of patterns is another issue. For example, you could search a pattern in a turbulent boundary layer using VITA technique. However, it is not until you survey how each captured VITA event is similar to the average that you can believe that the averaged pattern is really there. Otherwise, you are just looking at a suitably phase-aligned, random noise. Similar situations may happen in other flow fields.

Recently, we have attempted to cluster patterns in a parameter space. First, we retrieve zero-delimited segments out of a low-pass filtered turbulence signal, which we call *primary patterns*. The time constant of the filter is chosen depending upon the scales we are interested in. With many such primary patterns we calculate low-dimensional parameters such as width, height and centre of gravity. After suitably normalizing these quantities we plot every primary pattern in a parameter space. If the plot shows any cluster, then it can be a certain category of pattern which may have physicalsignificance.

We performed measurement of a turbulent boundary layer on a flat plate with 14 hot-wire probes taking streamwise velocity signals at different heights simultaneously. The Reynolds number based upon the momentum thickness is 2450, and the boundary layer seemed to be fully turbulent. We processed, with the present clustering scheme, one of the velocity signals and an artificial noise with the $-3/5$ power law. In this particular attempt we could not find any cluster for both signals. The fact may mean that naive parametric clustering cannot discriminate a turbulence signal from a pure noise, otherwise there may be no class of pattern in this particular velocity signal which is sufficiently more coherent than that in the noise. It seems that there only exist individual patterns with different sizes and shapes in a broad-band spectrum. Does a representative pattern exist in a fully developed turbulence?

It is certain that the present scheme can detect coherent structures in the transition region of a turbulence. However, we have doubt whether there exists essentially a method to *cognize* coherent patterns in a fully developed turbulence.

Evidence of low dimensional chaotic attractor in weather temperature measurements in Greece

G. Papaioannou and T. Bountis

Evidence is presented for the presence of a low dimensional chaotic attractor in weather temperature measurements in Greece. The time series is taken from daily (maximum) temperature data of the Greek Meteorological Service, measured over a period of 30 years, and at different locations. Using the Grassberger - Procaccia method, we find that the fractal (correlation) dimension of the observed chaotic attractor is slightly larger than 2 ($D_2 \geq 2.05$) and the embedding dimension id $d = 6$. It is interesting that the chaotic attractor appears to be independent of the location of the Weather Station. The possibility for short term prediction, using the methods of Farmer - Sidorowich and Casdagli, is also considered.

Experimental studies of time series obtained from transition to turbulence in pipe flow

T. Reimers and V. Wilkening

We describe an experiment, designed for a better understanding of the transition to turbulence in pipe flow. In our laboratory we realised pipe flow which remains laminar up to Reynolds number 7000.

In the sense of conventional linear instability theory, the Hagen-Poiseuille flow in a pipe is stable for all Reynolds numbers. From numerous experiments and simulations it is known, on the other hand, that a transition to turbulence takes place. The apparent discrepancy between both of these statements is bridged by recent abovo analytical theory of Boberg and Brosa (1).

The flow is disturbed at a certain location in a time periodic fashion by the electromagnetic Lorentz force. The resulting changes of the flow pattern are recorded by LDV and a TV camera (2). We observe the disturbance of the pipe flow depending on the Mach-Alven number

$$MA = \sqrt{\frac{\rho_0 v_0^2 Re^2}{\sigma_0 E_0 B_0 R_0^3}} \qquad (3)$$

(1) L. Boberg and U. Brosa, Onset of Turbulence in a Pipe, Z. Naturforsch. **43a**, 697-726 (1988).

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(3) U. Brosa, Disturbances in Pipe Flow Excited by Magnetic Fields, preprint.

Willem Van de Water

A dedicated computer for measuring fractal dimensions

The multifractal model describes the small-scale structure of turbulence as the reflection of interwoven fractal sets of singularities. Moreover, these fractal sets would exhibit multiscaling due to singularity- dependent viscous cutoffs. The problem in the experimental verification of the multifractal model is the poor convergence of the measured spectrum of generalized dimensions. We therefore have built a dedicate computer that is able to process at least one billion (10^9) points in real time.

Unstable periodic orbits in the parametrically excited pendulum

We analyze chaotic motion in an experiment on a parametrically excited pendulum in terms of unstable periodic orbits. This provides a useful quantitative comparison with results of a faithful numerical simulation. Despite the presence of experimental artefacts, simulation and experiment are in good agreement. The analysis of scaling properties of both chaotic attractors along these lines remains, however, incomplete. Periodic-orbit analysis fails to account for heir marginal hyperbolicity, and therefore fails to capture an important qualitative aspect of the chaotic dynamics in both experimentandsimulation.

Estimation of the persistence of strain from experimental time series recorded from cardiac muscle during ventricular fibrillation

H. Zhang, A.V. Holden, M. Lab, M. Moutoussis

The persistence of strain, which has been specifically used in fluid mechanics to measure the balance between the shear-dominated and vorticity-dominated flow at a point $\mathbf{x}$ of an incompressible velocity field, also suggests a quantity to characterize dynamical systems.[1] For any physical system governed by the equation

$$\frac{d\,\mathbf{x}}{d\,t} = u(\mathbf{x}, t), \quad \mathbf{x} = (x_1, x_2, x_3, \ldots, x_n) \tag{1}$$

we have

$$\frac{d\,\delta\,\mathbf{x}}{d\,t} = A(\mathbf{x}_0)\delta\mathbf{x}_0 \tag{2}$$

to describe the evolution of a small disturbance $\delta\mathbf{x}_0$ on the initial condition $\mathbf{x}_0$, where $A(\mathbf{x}_0)$ is called the Jacobian matrix,

$$A_{i,j}(\mathbf{x}_0) = \partial_j u(x_i, t)\,|_{\mathbf{x}_0} \quad i, j = 1, 2, 3, \ldots, n. \tag{3}$$

The equation (2) has the following solution

$$\delta\mathbf{x}(t) = \int A\,(\mathbf{x}_0)\delta\mathbf{x}_0 dt = T^t(\mathbf{x}_0)\delta\mathbf{x}_0 \tag{4}$$

where $T^t(\mathbf{x}_0)$ is a $n \times n$ tangent map matrix, and can be written as the multiple product of the $T^{\Delta t}(\mathbf{x}_i)$

$$T^t(\mathbf{x}_0) = T^{k\Delta t} = T^{\Delta t}(\mathbf{x}_{k-1}) \ldots T^{\Delta t}(\mathbf{x}_1) \cdot T^{\Delta t}(\mathbf{x}_0). \tag{5}$$

When $\Delta t \to 0$, we have

$$T^{\Delta t}(\mathbf{x}_i) = A(\mathbf{x}_i)\Delta t \tag{6}$$

If the eigenvalues of the matrix A are complex conjugate pairs $\lambda_i = \alpha_i + \beta_i$, then the persistence of strain σ^2 is defined as

$$\sigma^2 = \mathrm{Tr}A^2 = \sum_{i=1}^{n} (\alpha_i^2 - \beta_i^2) \tag{7}$$

We calculate the persistence of strain from experimental time series $x_i = x(i\tau)$, $i = 1,2,...,N$ by the following steps:

i) Reconstruct the dynamical attractor from the time series in phase space with embedding dimension d_E. [3,4] A point in the phase space is

$$\mathbf{x}_i = (x_i, x_{i+1},...,x_{i+d_E-1}), \ i = 1,2,..., N-d_e+1 \tag{8}$$

ii) Divide the evolution of the trajectory by a series of small time intervals Δt, and examine the evolution in Δt to obtain the tangent map by a least-squares fit. Suppose $\mathbf{x}_j (j = 1, 2,..., m)$ is one of the m neighbours contained in a ε ball centred at the point $\mathbf{x}_i$, e.g.

$$\| \mathbf{x}_i - \mathbf{x}_j \| \le \varepsilon, \ j = 1, 2, ..., m \tag{9}$$

after a time interval Δt, $\mathbf{x}_i$ evolves into $\mathbf{x}_i'$, $\mathbf{x}_j$ evolves into $\mathbf{x}_j'$, the small distance between them evolves by

$$\mathbf{x}_j' - \mathbf{x}_i' = T_i^{\Delta t} (\mathbf{x}_j - \mathbf{x}_i) \tag{10}$$

Where T_i is the tangent map matrix, and takes the form

$$T_i^{\Delta t} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & & 1 \\ a_1 & a_2 & & \dots & a_{d_E} \end{bmatrix} \tag{11}$$

We can obtain a_k by a least-square fit [2]

$$\sum_{j=1}^{m} \left(\sum_{k=1}^{d_E} a_{k+1}(x_{j+km} - x_{i+km}) - (x_{j+km}') \right)^2 = \text{minimum} \tag{12}$$

(iii) Calculate the persistence of the strain from the tangent maps.

This software, together with the software for the reconstruction of attractors and the estimation of the Lyapunov exponents has been applied to the recordings of the electrical and mechanical activity obtained during experimentally induced ventricular fibrillation in the isolated perfused rabbit heart.

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Time series analysis of electroenecphalograms

A. Babloyantz

The time series analysis of non-linear physiological processes such as electro-encephalographic recordings (EEG) have provided a wealth of information about the dynamics of brain activity, which are difficult to assess by other techniques.

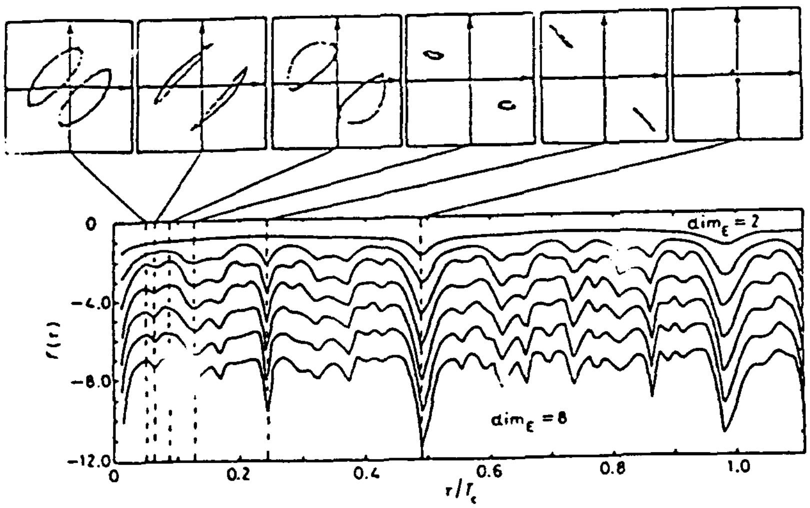
Among these the "recurrence plot" is a powerful tool for revealing the presence of drift or periodicities in a given time series and is easily obtained in the framework of a nonlinear dynamical analysis. When this analysis is applied to the EG recorded from a patient suffering from Creutzfeldt-Jakob disease one observes the presence of slow periodicities of the order of 58 sec. We suggest that the recurrent plots are powerful tools for discovering hidden periodicities of EEG as well as assessing the degree of stationarity of brain activity.

Most physiological attractors are non uniform. We introduce a new approach for analyzing the dynamical properties of non-uniform attractors. The attractors are viewed as a juxtaposition in phase space of several subattractors with different chaotic signatures. Therefore they could be analysed by adapting the existing algorithms. Moreover, a systematic probing of the semi-local dynamical properties leads to their probability distribution on the attractor. This approach provides new insight in the analysis of EEG and shed some light on the nature of cortical events.

Calculation of optimal embedding parameters for Takens' delay time coordinates with global geometric and local dynamic methods

Th. Buzug and G. Pfister

For the characterization of time series measured on real physical systems (e.g. the Taylor-Couette experiment) it is necessary to calculate dynamical variables as entropies, Lyapunov exponents and dimensionalities (1). This requires a proper reconstruction of the attractor in phase space, so that all relevant components of the dynamic are extracted from a single time series with Takens' delay time coordinates. Though the principle ideas are clear the practical procedure of the reconstruction has to overcome the difficulties which arise form short time series with restricted resolution. Two methods are presented to obtain the best delay time and a sufficient embedding dimension. The first procedure, which leads to a so called *Fill-Factor* $f(\tau)$, uses purely geometrical considerations and guarantees a maximum distance of trajectories (2). The figure below show the *Fill-Factor* for a measured 2-Torus and some corresponding Poincaré sections for proper delay times (maxima of $f(\tau)$) and unsuitable delay times (minima of $f(\tau)$).



Fill Factor vs. normalized delay time for a 2-torus (T_C is the mean reciprocal value of the dominant frequency obtained from the power spectrum)

Deterministic properties and fractal dimension of the volcano source system from volcanic tremor records

W. Brüstle

Abstract not received.

Local dimensions for detailed studies of continuous and discrete systems

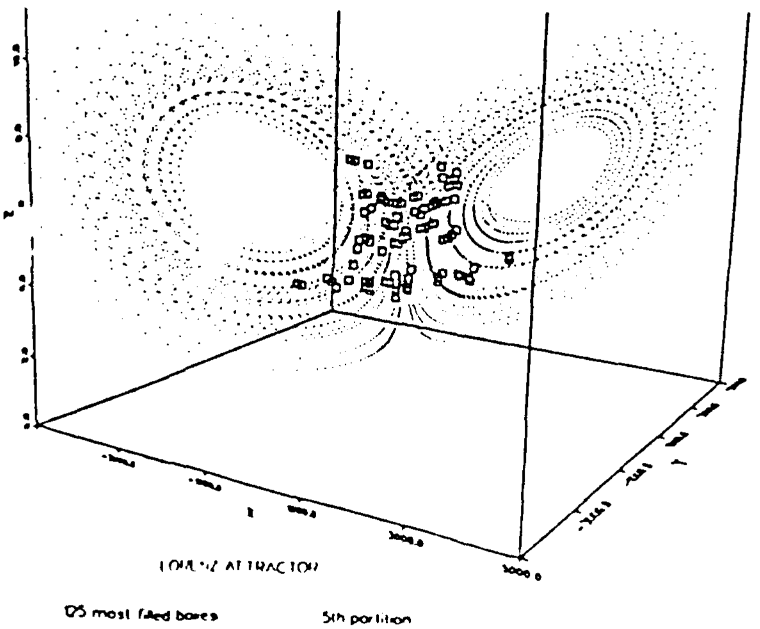
J.G. Caputo, G. Gouesbet, B. Maheu, S. Meunier-Guttin-Cluzel

There are two main ways of characterizing the chaotic behaviour of dissipative dynamical systems. The first one is to study the probability distribution of points in phase-space (fractal dimension, generalized Renyi dimensions) and the second is to look at the probabilities in trajectory space (generalized entropies, Lyapunov exponents).

We have studied the distribution of scaling exponents in phase-space (i.e. local dimensions) as well as in trajectory space (i.e. local entropies). Using the Lorenz system as a paradigm we showed that these scaling exponents vary continuously along the orbit, leading to well defined regions on the attractor.

As a consequence, the distribution of scaling exponents in phase-space allows to locate the most densely populated areas and leads to an estimation of the Renyi dimension, D_{∞} 1.97, for the Lorenz attractor. In a similar way we outline the regions with high or low rate of divergence of trajectories and on results confirm those of Nese [1].

We conclude that local dimensions, avoiding average over the whole attractor, yield detailed information about a chaotic system and particularly about its dynamics along a given orbit.



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Oscillations and onset of chaos and related bifurcation results for a nonlinear oscillator with possible escape to infinity

E. Del Rio

Abstract not received.

COREX: user friendly program for estimating correlation exponent of EEG

Ivan Dvorak, Jiri Wackermann

Characterization and classification of EEG recordings by estimating its correlation exponent ν attracts still growing popularity. For standardized estimation of this invariant we developed a user-friendly program based on Grassberger-Procacia algorithm.

The algorithm is improved in order to compensate various sources of bias. A downward bias, detected when correlation exponent of a white noise is estimated, is compensated by eliminating the impact of "edge effects" by using $\log C(\varepsilon)/\log(\varepsilon(2-\varepsilon))$ plot in the GP algorithm [1]. An upward bias may be detected when the correlation exponent of a low dimensional deterministic signal is estimated; it increases with increasing reconstruction dimension M [2]. In CODEX this bias is compensated by liner extrapolation of the curve in the saturation area of the ν/M plot to the crossing with the axis of the first quadrant. Value of ν corresponding to this crossing is then regarded as the correct estimate of the correlation exponent.

The program operates in two options. In local option, the state space representation of the supposed underlying dynamical system is reconstructed by the Takens method of delayed coordinates for a EEG recording from the chosen location. Time delay is chosen as the first zero crossing of the signal autocorrelation function. Estimated value of the correlation exponent brings information about local coordination of the brain processes in the chosen site.

Global option uses selected groups of channels (modes) for multi-channel state space reconstruction. Obtained value of the correlation exponent characterizes the degree of binding between processes taking place within various parts of the cortex.

Estimation of the correlation exponent is performed by an iterative step procedure that chooses in random pairs of points in the state space and calculates their distances. In each step about 10000 pairs are chosen. An average is calculated form results obtained in each step. When the variance of ν falls under limit prescribed beforehand, calculation is terminated. The whole process is fully automated and requires no user-driven steps.

At least one minute long record sampled by 128 Hz (approximately 120 00 samples) is required for reliable estimates. Up to 16 channels of EEG recording can be scrutinized. Varied types of registration, montage and reference are accepted.

COREX runs on IBM PC/AT true compatibles with 87 coprocessor under DOS 3.xx. Calculation of one value from a single one minute record lasts several seconds.

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Estimating dimension with confidence

Kevin Judd

The irrefutable identification of a low dimension attractor from a time series could be an important step in the analysis of a complex system. For this reason, the estimation of the correlation dimension of a supposed attractor represented in a time series has become a frequently applied technique; and a frequently misapplied technique. The principle problem is that although it is easy to calculate an estimate of the dimension, no error of the estimate is provided, which makes the estimate meaningless. A new algorithm to estimate the dimension is presented that provides a form of confidence interval for the dimension.

The most common algorithms for estimating the correlation dimension are variants of the Grassberger-Proccacia method of estimating the asymptotic slope of a correlation function. For a finite amount of data the procedure is to select a scaling region and estimate the slope there. This results in three problems: the selection of a scaling region is always subjective, the scaling region is rarely of uniform slope but is curved, and the estimate of this slope includes no estimate of the error of the dimension estimate.

The proposed new algorithm works directly from the distribution of the interpoint distances $p(e)$, rather than from a correlation function. The distribution is divided into two parts: an upper part that provides only information about the finite size and large-scale properties of the attractor, which are irrelevant; and a lower part that provides true scaling information. The choice of this division is not particularly critical. By assuming a simple model of the fractal structure of the attractor, it is shown for the lower part of $p(e)$ that

$$p(e) = \exp(de)\, q(e),$$

where d is the dimension and q is a polynomial with degree equal to the topological dimension of the attractor. By making a maximum likelihood fit of the parameters of the distribution onto some data, one obtains not only an estimate of the dimension but also an estimate of a confidence interval for the dimension.

The algorithm has been tested on artificial data of known dimension and has performed well.

On the determination of dimension from sampled data

E. Kreuzer

Abstract not received.

Distinguishing between deterministic chaos and random noise and diagnostics of chaotic systems from the time series

P.S. Landa and M.G. Rosenblum

Let us consider the system of the unknown structure ("black box") which is influenced by the unmeasurable noise, and there is some chaotic process $x(t)$ at the system's output. The main question in "black box" investigation is: if the system investigated is a noise transformer or a self-oscillation system, generating dynamical process, to which noise is added (identification problem). In the latter situation there arise the following problems for a researcher:

1. To extract from the process observed the deterministic (i.e. generated by the dynamical system) component.
2. To obtain the dynamical system model and evaluate the minimal number of variables required.
3. To determine the system state (the set of parameters) by the appropriate processing of $x(t)$ (systems diagnostics).

It is necessary to mention that using the term "noise" here we mean stochastic process created by the big amount of independent sources. The dimension of this process is much higher than the dimension of a signal which can also be stochastic. In its turn, when w say "dimension of the process" we mean the dimension of the attractor in the phase space of dynamical system, generating this process.

The time series preprocessing technique is proposed, which allows to filter off noise and simultaneously evaluate the embedding dimension of the attractor and thus reduce computer expenditures. This technique is based on the transformation of the Takens phase space and includes the transition to the new basis of smaller dimension and scale transformation. The new basis is chosen according to Karhoonen - Loeve theorem [D.S. Broomhead, G.P. King, Physica D. - 1986. -V.20 -P.217-236.] or Neymark algorithm [P.S. Landa, M.G. Rosenblum, Physica D, 1991, to appear]. These algorithms ar compared and the superiority of the second one for investigation of inhomogeneous attractors is demonstrated. Calculation of the attractor correlation dimension in the transformed space allows to distinguish reliable coloured noise and dynamical process, while calculation in the Takens space may lead to erroneous conclusions. An additional identification criterion - the mean one step prediction error - is proposed. The example of the solution of Rossler system and the noise with the same power spectrum is considered.

Comparison of different quantitative characteristics of chaotic motion for bubble oscillations in the sound field

P.S. Landa and M.G. Rosenblum

Different characteristics can be used for quantification of chaotic motion. Comparison of these characteristics on the example of acoustic turbulence is performed. To simulate this phenomenon, the model of single bubble oscillations in the sound field is used [J. Holzfuss, W. Lauterborn Proc. of the 13th Inter. Congress on Acoustics, Belgrade, 1989, V.1, P. 127-130]. For imitation of a real experiment some noise is added to the periodical term which describes the driving sound field. The intensity of noise was approximately 10 times less than that of the sound field. This non-additive noise is sufficiently eliminated by iterative filtration. The filtration is based on the algorithm of choosing the Neymark well adapted basis. The correlation dimension calculated form the filtered time series practically coincides with the one obtained in the absence of the noise term in the model, while the value obtained from the noisy signal is stronglyunderestimated.

Such characteristics of chaotic motion as generalized dimensions and Lyapunov exponents are compared with Shannon and normalized entropies which were introduced for quantification of degree of order by Yu. L. Klimontovitch. The dependencies of these characteristics versus frequency of the driving sound field and vapour pressure in the bubble are calculated.

Observation of stochastic resonance in a bistable system with periodically modulated noise intensity

M.I. Dykman, D.G. Luchinsky[1], P.V.E. McClintock, N.D. Stein and N.G. Stocks

Previous studies of *stochastic resonance* (SR) in which, remarkably, a weak periodic signal in a noisy bistable system can be amplified by the introduction of additional external noise, have related to the case where the periodic signal and the noise are additive. We now report the preliminary results of an investigation in which the signal and noise are introduced *multiplicatively*. We treat the case of an overdamped Brownian particle moving in an asymmetric bistable potential

$$\dot{q}(t) + U'(q) = \left[\frac{1}{2} A \cos(\Omega t) + 1 \right] \xi(t) \qquad (1)$$

where $\xi(t)$ represents Gaussian white noise, periodically modulated at frequency Ω.

The system has been studied through the analysis of digitized time series derived form an analogue electronic model of (1), and theoretically. It is found, first, that SR does exist for periodically modulated noise intensity, and that the experimental data are in good agreement with theoretical predictions based on linear response theory (LRT)[2]. Secondly, this new form of SR exhibits characteristic features that are strikingly different from those observed previously for conventional SR. In particular, the periodic signal vanishes for the special case of equal well-depths ($\lambda = 0$), which is where the conventional SR signal passes through an extremely sharp maximum.

These and other unusual features of SR under periodically modulated noise intensity will be presented and discussed.

1. Permanent address: All-union Research Institute for Metrological Service, 117965 Moscow, USSR.

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Fractal-dimension analysis of coupled maps

Antonio Politi

In the past years a rather complete understanding of low-dimensional chaos has been accomplished. Various general relations among dynamical invariants have been established and powerful algorithms have been developed to extract information directly from experimental as well as numerical data. Much more preliminary is the comprehension of spatio-temporal chaos, where reliable numerical investigations require to handle huge data sets, and geometrical intuition is of very little help.

Among the diverse aspects characterizing the complex evolution of spatially extended systems, here we are mainly interested in the role of temporal and spatial correlations in the invariant measure in presence of fully developed spatio-temporal chaos. This is done by refining the standard techniques used to estimate the fractal dimension. This method, implicitly based on the determination of mutual information, is more powerful than the standard computation of suitable correlation functions. We limit ourselves to the case of a 1-d lattice of coupled maps, in order to exploit the discreteness of the space variable which allows for a considerable reduction of computer time. Following Grassberger, we have projected the probability distribution, of an - in principle - infinite chain, onto (temporal and spatial) embedding spaces of increasing dimensions. The resulting coarse-grained fractal dimension is shown to exhibit an increasingly slow convergence, for increasing the dimension of the embedding space. Accordingly, the presence of strong correlations concentrating the invariant measure around some lower-dimensional manifold has been conjectured.

To decide whether the difficulties encountered in the dimension-estimate are either intrinsic of the invariant measure, or follow [illegible]ially from [illegible] use of an insufficient n[illegible]ber of [illegible]ts, we have also performed a[illegible]lative ana[illegible]. Namely, orthogonal decomposition has been applied locally to [illegible]mine the [illegible]ge thickness of the measure along the principal axes. More p[illegible]sely, 200 refe[illegible] points have been randomly chosen, and boxes of different si[illegible], have been set around them. As a result, anomalously small thicknesses have ben detected, confirming that the slow convergence of the dimension is due to the presence of strong correlations.

Finally, an attempt has been made to find possible relations between the clustering of points in phase-space and the existence of localization phenomena in tangent space. To this aim, a transfer matrix approach has been developed to determine the localization length of the Liapunov vectors. The different vectors have been also projected onto suitable embedding spaces and the resulting dir[illegible]ns compared.

Wavelet analysis of fully developed turbulence data at high Reynolds number

V.A. Sabelnikov, A.A. Praskovsky and D.A. Usikov

The wavelet analysis was used for direct measurements of the scaling exponents characterizing the local multifractal behaviour of the velocity fluctuations at inertial-range scales.

The velocity signals analysed were obtained in the return channel of the CAHI large wind tunnel. The Taylor scale based Reynolds number was $R_\lambda \approx 3100$ and the extent of the inertial range was more than two decades. The integral turbulence scale was 4.8m and the Kolmogorov scale was $\eta = 0.4$mm. The measurements were carried out using single-wire and X-wire probes with characteristic sizes $\ell \leq 0.5$mm. The spatial resolution was rather good ($\ell/\eta \leq 1.25$).

The main efforts were directed to obtain the most reliable results. In particular it was shown that the wavelet transform had to be averaged over a length of the order of the highest inertial-range boundary to decrease scaling exponent measurement errors.

The scaling exponent histograms for three velocity components were obtained and the measurement errors were analysed. It was shown the exponent values α varied form -0.3 up to 0.9 and the average and the most frequent exponents were close to the Kolmogorov value 1/3. The measurements errors rised with increasing the difference between the exponent α and the value 1/3, hence the exponents measured $\alpha < -0.1$ and $\alpha > 0.6$ were doubtful.

Using multivariate data analysis to compare time series

G.P. King and C.T. Shaw

When laminar flows become turbulent, they may follow several so-called *routes to chaos*. One way of determining the routes is to measure the temporal variation of flow velocity at a set of given positions in the flow, and then to compare the frequency spectra derived from these time series. We have been investigating the flow behind a finite cylinder at a fixed Reynolds number, where a Von Karman vortex street created by the cylinder becomes a turbulent wake further downstream of the cylinder. Measurements of the fluctuations in the flow have been measured with a hot-wire anemometer, and several regions within the wake of the cylinder can be detected where the spectra are similar.

Multivariate data analysis techniques have been developed that enable sets of data, in this case frequency spectra at a set of points, to be classified. Such techniques are widely used in the Social Sciences to investigate the similarities and differences between the characteristics of various groups. For example, the behavioural characteristics of males and females can be compared. Also, these techniques are now used by astronomers to classify the spectra obtained from stars. Taking this as an example, it should be possible to use the techniques to classify the time series determined from a flow.

In an attempt to see whether some form of automatic classification of the spectra in the wake of the cylinder can be produced, we have used Principal Components Analysis and Cluster Analysis to group the spectra automatically. The results of this have then been compared to a manual classification of the spectra. There is good agreement between these two classification methods and we hope to be able to use the automated techniques in future to investigate a variety of transitional flows.

Effect of quasi-monochromatic noise on nonlinear systems

M.I. Dykman, P.V.E. McClintock, N. D. Stein and N.G. Stocks

Quasi-monochromatic (narrow band) noise[1], with a power spectral density of the form

$$Q(\omega) = 4\Gamma T\,[\,(\omega^2 - \omega_0^2)^2 + 4\Gamma^2\,\omega^2]^{-1} \qquad (1)$$

provides a much better description of the fluctuations that occur in many real systems (e.g. large molecules, microwave cavities, engineering structures) than earlier idealisations such as white noise or exponentially correlated noise. As expressed in (1), the fluctuations correspond to the thermal noise of an underdamped harmonic oscillator of eigenfrequency ω_0 and damping constant $\Gamma \leq\leq \omega_0$ coupled to a thermal bath at temperature T; but the driving noise need not be thermal in origin, and may in principle come from any quasi-white noise source of intensity T. Quasi-monochromatic noise (QMN) may also be perceived as offering a (much-needed) bridge between the two major traditions - deterministic an stochastic - of nonlinear science. This is because, in different extreme limits, (1) reduces either to a pure periodic (deterministic) driving force, or to the white noise (stochastic) idealisation of Einstein.

We have investigated[2] the effect of QMN on overdamped bistable and monostable systems through the analysis of digitized time series from analogue electronic models. (The effect of QMN upon underdamped systems is also under investigation[3], in UCL). The response of overdamped systems to QMN is found to differ from that to more conventional forms of noise in a number of important respects. For example, in the case of a bistable system, fluctuations about one of the attractors can readily pass across the central potential maximum *without leading to a transition* of the system to the other attractor - quite unlike the cases of excitation by white or exponentially correlated noise. It is also found hat, for systems fluctuating in a single well potential, the probability distribution is *independent of the shape of the potential.* It will be shown that these, and other remarkable features of QMN observed in the experiments, are in good agreement with theoretical predictions.

The significance of these results, and the relevance of QMN studies to the application of nonlinear science to the real world, will be discussed.

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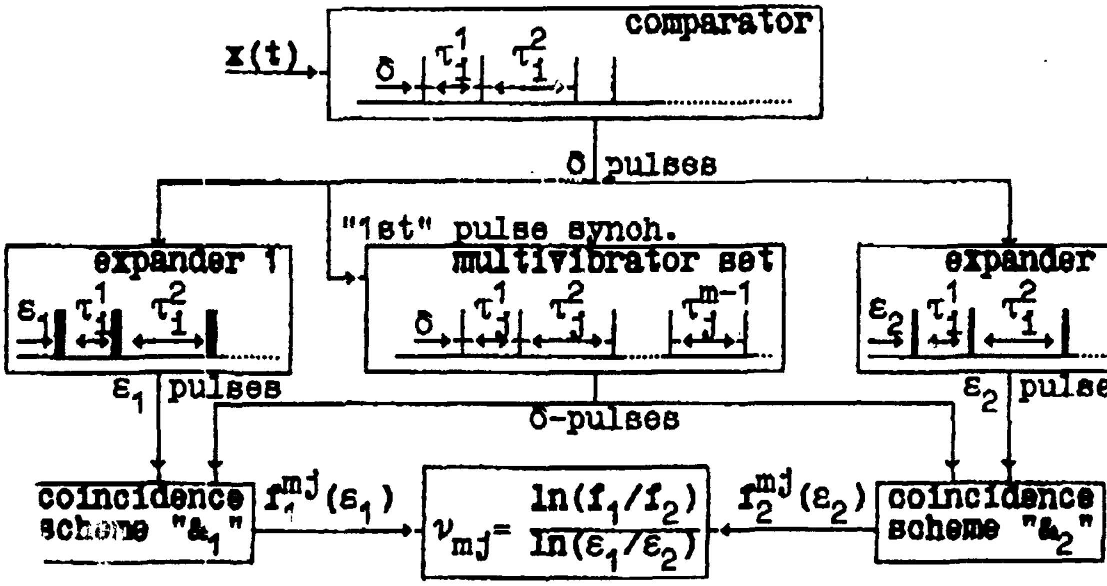
A technique for measuring fractal dimensions from time series in a real time scale

A. Namajunas and A. Tamasevicius

Various digital algorithms have been proposed for determining fractal or integer dimensions of chaotic systems from time series data. Several attempts have been made to develop analog techniques for measuring dimensions without the need of digital computing equipment [1, 2].

Our previous work [2] is restricted to the phase space dimension of local embedding, which is an integer. Here we present similar electronic technique modified for estimating the pointwise correlation dimension (fractal) from a single observable $x(t)$ in a real time scale.

Figure shows a sketch of the experimental setup.



The coincidence frequency (the number of the coincidences per unit time between the sets $\{\tau_i^1,\ldots,\tau_i^{m-1}\}$ and $\{\tau_j^1,\ldots,\tau_j^{m-1}\}$) is

$$f^{mg}(\varepsilon) \sim \sum_i \theta\left(\varepsilon - |\tau_i^{m-1} - \tau_j^{m-1}|\right) \sim \varepsilon^{\nu(mj)}.$$

The correlation dimension ν is defined as $\nu(mj)$ at large embedding dimension m. This technique is limited to the pointwise fractal dimensions (the sum is taken over "1", but not "j"). Nevertheless, the reference point "j" can be easily chosen in the neighbourhood of the most visited parts of the attractor by means of adjusting τ_j and selecting the maximum value of f^{mj}.

Dimension of spatial field distributions of nonequilibrium media

V.S. Afraimovich, A.B. Ezersky, M.I. Rabinovich, M.A. Shereshevsky and A.L. Zheleznyak

The state of a nonequilibrium medium at every moment of time t is an instantaneous spatial distribution - a snapshot - of a field (fields) that forms an "observable". In simplest case, when the medium is described by gradient model, the observable loses its time dependence as $t \to \infty$, whereas the spatial distribution may be regular or chaotic. In a more general case, the temporal dynamics is also complicated.

To formalize the characteristics of spatial field distributions we use such a notions as dynamical system with d-dimensional time, translational invariance, snapshot dimension, etc.

Let us introduce the phase space of the system - a set B of continuous (vector-) functions $U(x)$, $x \in \mathbb{R}^d$, with a distance on it. Let each d-dimensional vector $\alpha = (\alpha_1,...,\alpha_d)$ correspond to the translation map $T^\alpha : B \to B$ that is given by the expression $T^\alpha(U(x)) = U(x - \alpha)$. Thus, the action of the group $\mathbb{R}^d$ on B determines the dynamical system with d times - the translational dynamical system. On the other hand, if the process under study is such that knowing the initial state one can unambiguously determine the subsequent states at any moment of time, then a subgroup of evolutionary operators $\{S^t\}$ also acts on B, i.e. the evolutionary dynamical system is determined as well. The behaviour of the trajectories of these two systems in the common phase space B gives a full mathematical description of the spatio-temporal properties of the nonequilibrium medium of interest.

On the base of this approach we introduce the rigorous definitions of snapshot characteristics - limit capacity, hausdorff dimension and dimensions which depend on invariant measure, supposing that these characteristics describe a set of limit points along the trajectories of translational dynamical system [1].

Further, generalizing the Takens approach to the systems with d-dimensional time, we propose the algorithms for the calculation of the dimensions of spatial fields distributions.

The constructed numerical procedure for the computing of the correlation dimension of two dimensional snapshots directly from the experimental data is presented. This procedure has been employed for the description of temporal intermittent chaotic dynamics of parametrically excited capillary ripples [2].

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Using neural nets to look for chaos

A.M. Albano, A. Passamante, T. Hediger and Mary Eileen Farrell

Backpropagating neural networks have been used to reconstruct the attractors of some low-dimensional chaotic systems from noise-corrupted scalar time series of one of the system's variables. The performance of the network is judged *not* by its ability to predict a time series that approximates one that can be calculated from equations of motion using the same initial values, but rather by its ability to construct an attractor that is visually similar to, and has the same correlation dimension as, the attractor obtained from the equations of motion.

Using data calculated form the logistic equation and the Hénon map to which various amounts of random Gaussian noise have been added, the expected attractors have been reconstructed using as few as eight input data points that are noise-free except for the limitations of single-precision calculation, and as few as sixteen points of data with a signal-to-noise ratio (signal variance/noise variance) of as low as 10 db. The correlation dimensions of the reconstructed attractors for both noise-free and noisy cases agree remarkably well with those of the attractors obtained form the dynamical equations. This latter observation is remarkable in view of the fact that filtering of chaotic data is known to increase the dimension of the signal's attractor.

Time series analysis applied to a discontinuous dynamical system

Chris Budd and Harbir Lamba

In this talk we shall analyse the time series obtained from an impact oscillator. Here we study the forced motion of a mass on a spring impacting a rigid obstacle. It is possible to measure either the velocity and time of each impact or the position and velocity of the mass at equally space time intervals. These observations lead naturally to discontinuous Poincaré maps relating present to future pairs of points. The reconstruction of the dynamics of the original system from the time series is difficult because of the discontinuities in the maps but numerical evidence indicates that the Takens reconstruction theorem is still applicable.

We present the results of the application of several time series analysis algorithms to the observations described above and compare their effectiveness in reconstructing the asymptotic dynamics of the original system. We compare the results of the application of these algorithms with similar results obtained from a dynamical system related to the impact oscillator but with smooth behaviour.

Bifurcation diagrams of Duffing-type oscillators derive from circle maps

G. Eilenberger and K. Schmidt

We show how the behaviour of a class of damped driven nonlinear oscillators of type

$$\ddot{x} + 2r\dot{x} + V(x, \Omega t) = 0$$

is asymptotically described (for small r and small $\Omega = \pi/T$) by circle maps of the type

$$\varphi_{n+1} = A_1 T = + C + r^{-\kappa}\, \Omega^{\lambda}\, A_{@}(1 + B\, e^{-rT} \cos \varphi_n).$$

We derive these maps and determine the constants A_1, A_2 and B for special potentials V. We argue that these special cases contain all the information needed to treat the general case.

Deterministic chaos versus random noise in rainfall time series

Paolo Ghilardi

We look for chaotic behaviour in several time series from rain gages located in Northern and Central Italy. We make our tests considering a wide range of time scales, using different sampling times: from the very short time scale characterizing heavy storms, to the monthly time scale, needed in assessing the seasonal behaviour.

In order to check for chaotic behaviour, the Grassberger-Procaccia correlation integrals are computed, together with the power spectra and the fractal dimensions of the time series. We implemented our algorithm for the computation of the correlation integrals, modifying the original Grassberger-Procaccia procedure as Theiler (Physical Review A, Vol. 34 (1986) 2427) and Smith (Physics Letters A, Vol. 133 (1988) 283) suggested. Analyzing short, heavy storms, the slope of correlation integrals do saturate to a low value, similarly to what happens in analyzing data form a low dimensional chaotic system, but the values of the correlation dimension are less than 2.0 in some cases, thus contradicting the Kaplan-Yorke conjecture.

Employing some standard methods the self-affinity of these time series is assessed, and values of the scaling exponents are found. A standard spectral analysis is also performed, finding how these time series show power-law spectra.

The rainfall time series we analyze from heavy storms are then self-affine, and are characterized by power law spectra and by a finite value of the correlation dimension. Osborne, Provenzale et al. (see, e.g. Physica D, Vol. 35 (1989) 357) found coloured stochastic noise to be characterized by power law spectra, self-affinity, and by a finite value of the correlation dimension. They also found the relations between the spectral exponent, the scaling exponent and the correlation dimension characterizing that class of stochastic process: it is interesting to note how these relations are well satisfied using the parameters we compute from rainfall time series. Then, records from heavy storms display similar characteristics to those from stochastic systems with power-law spectra.

We conclude that we cannot asses the deterministic chaotic behaviour of storm rainfall, even if the correlation dimension saturates to low finite values.

Indicators on space complexity in extended chaotic systems

F.T. Arecchi, G. Giacomelli, P.L. Ramazza dns S. Residori

Video showing the dynamics of topological defects/vortices with spiral arms, dislocations in optics.

Time series near a homoclinic bifurcation

James Glover

By analysing the statistical properties of a transformation of a time series we are able to detect if it passes near a fixed point, the number of such points and their eigenvalues and indexes.

Interpretation of aperiodic time series: distinction between strange chaotic and strange non chaotic attractors

J. Brindley, T. Kapitaniak and J. Wojewoda

Chaotic behaviour of nonlinear systems is characterized by the exponential divergence of neighbouring trajectories near to or on an attractor, and by the 'strangeness' of the attractor itself we mean having a geometrical configuration not definable by a finite number of points, smooth curves or surfaces.

It is now well established however that some systems are characterized by non-chaotic strange attractors. In this case the attractor is geometrically strange, but neighbouring trajectories do not diverge exponentially.

As both types of strange attractors look very similar (both have fractal structure), the value of Lyapunov exponents seems to be the only quality which allows us to distinguish these classes.

For systems in which equations of motion are explicitly known and the linearized equations exist there is a straightforward technique for computing a complete Lyapunov spectrum. However, for most of the experimental systems, the equations of motion are not known or are in the form for which the liberalized equations do not exist (for example as described below, a system with dry friction). In this case Lyapunov exponents are estimated on the basis of the monitored long-term time series, and the attractor recontracted by well-known techniques with delay coordinates.

In this paper we first consider two examples of equations having both strange chaotic and strange nonchaotic attractors. In these cases we have computed Lyapunov exponents twice, once form the equations of motion and once from time series. We show that, when we know the differential equations which describe our system, we can distinguish between strange chaotic and strange nonchaotic attractors. This distinction cannot be made when the Lyapunov exponents ar estimated from a single time series, as this method gives positive Lyapunov exponents for both cases of strange attractors. This surprising result is because the whole procedure of reconstructing the attractor explores the aperiodicity of time series and not the explicit dependence on initial conditions.

Using the information dimension of the projection of the attractor onto a plane, it is possible in some cases to distinguish between strange chaotic and strange non chaotic attractors. According to The Kaplan-Yorke conjecture the information dimension of strange nonchaotic attractor in the four-dimensional phase space should be less than 2.

Finally we use the information dimension criterion to follow up a distinction between both kinds of strange attractors in an experimental mechanical system with dry friction which has been excited by two independent periodic forces. Depending on the system parameters we have found that the friction force has strange properties. Based on the estimation of information dimension it appears that friction force can be considered as chaotic function of sliding velocity.

Nonlinear time series forecasting, adaptive networks and numerical optimisation strategies

David Lowe and Andrew Webb

In the study of naturally occurring phenomena with the prior intention of *forecasting* one often only has indirect access to the underlying process by noisy observations of specific states of the system under study. Without detailed prior knowledge and physical insight it is rare to have access to a reliable model of the process (as for instance in asset price modelling in stock markets, or macroeconomic modelling). Even with such a model it might be impractical to specify the boundary conditions sufficiently to be able to predict its future behaviour given the model and past behaviour. The alternative is to analyse the statistics and dynamics of the observation sequence and try to infer some knowledge about the future evolution of the system. The problem with this latter approach is that Nature tends to produce very complicated sequences. Until recently, forecasting theory has developed to assume that the observation sequence is one particular realisation of a random process where the randomness arises from many degrees of freedom interacting linearly. However, the contemporary dynamical systems approach is such that an observation may be generated by a deterministic system with only a small number of degrees of freedom, but interacting *nonlinearly*. Thus, one problem in trying to construct a model of the underlying *generator* of the data is that the model itself should be nonlinear.

Adaptive ('neural') networks, currently so popular because of the historical link with neuromorphic computing ideas, may be viewed as flexible parametrised nonlinear template models, where the parameters may be adjusted according to available data, without recourse to an explicit physical model (although incorporating prior knowledge is always an advantage). Such networks perform functional interpolation to the input-output mapping function in a high dimensional outer product space. These networks are capable of performing a *nonlinear* transformation between input and output spaces (which are implicitly considered to be Euclidean), and thus may be suitable in the analysis of patterns (such as time series) generated by an underlying nonlinear model, particularly if the generator is *static* and *smooth*. Thus, adaptive networks could be an effective tool in the analysis of nonlinear time series by attempting to 'curve fit' the generator of the data, rather than the data itself. Of course curve fitting is nontrivial even in one dimension. The principle is well established but the practice and numerical implementation can be fraught with problems. Acceptable generalisation performance relies on sufficient data being available to estimate the parameters and on the existence of nonlinear optimisation techniques to allow the fitting process to be carried out efficiently. In recent *classification* problems in pattern recognition, it has been identified that the various layers of which the adaptive networks are composed, perform functionally distinct roles - specifically *feature extraction* and *discriminant analysis*. It is natural to consider the optimisation of these two roles as evolving on two distinct time scales where the discriminant analysis is *slaved* (in the synergetic sense) to the feature extraction dynamics.

In this paper we consider the utility of contemporary networks in nonlinear time series forecasting. Explicit demonstrations will be presented showing how the 'artificial neural' networks approximate the generators of the time series (as opposed to the time series itself as in conventional curve fitting). A hybrid nonlinear/linear optimisation scheme for such networks with two time scales will be described in the context of generalisation and parameter adaptation, with strengths and weaknesses highlighted. Using a range of example nonlinear time series (synthetic and real, single and multichannel) the limitations of network models will be discussed. In addition, a network version of 'nonlinear principal components analysis' will be introduced.

Geometry from data

Alistair Mees

Given data assumed to come from a dynamical system, it would be useful to be able to do more than produce a black-box model for prediction. Understanding the geometry of attractors or of state spaces (or manifolds) is important too. For example, detecting that a region of the space is folded or branched by the dynamics would be helpful in building qualitative models, or analytic models and also in interpreting data directly. This contribution discusses the use of tesselations and triangulations in this area as well as in production of predictive models.

Estimation of Lyapunov exponents form chaotic time series based on approximations of the flow map by radial basis functions

Ulrich Parlitz

Algorithms for computing Lyapunov exponents from experimental time series are presented that are based on local approximations of the flow map (and its Jacobian) in embedding space in terms of radial basis functions (RBF's). The RBF's (Hardy's multiquadrics, Gaussian functions, ...) are centered at neighbouring points of the reference points along the orbit. First investigations using an interpolation scheme have revealed very promising results for clean and weakly polluted data even for a time series originating from a strange attractor with three positive Lyapunov exponents (J. Holzfuss and U. Parlitz, *Lyapunov exponents from time series*, Proceedings of the Conference *Lyapunov Exponents*, Oberwolfach 1990, eds. L. Arnold, H. Crauel, J.-P. Eckmann, in: *Lectures Notes in Mathematics*, Springer Verlag.)

When the data are strongly polluted by noise, however, these interpolation methods yield unsatisfactory results and algorithms using approximations of the data points are needed to estimate reliably the Lyapunov exponents of the time series. Using ideas and methods from regularization theory (Tikhonov-Phillips regularization, filtered singular value decomposition,...) such approximation schemes have been implemented and will be tested with respect to the choice of the RBF, the smoothing method, and characteristics of the data. The question how to choose the free stiffness nd regularization parameters occurring in the algorithms is addressed and leads to cross-validationtechniques.

The possibility to use the presented methods for related tasks (nonlinear prediction, noise reduction, determination of unstable orbits, controlling chaotic systems) will be discussed.

Evaluation of time series from forced excitable systems

P. Pokorny and M. Marek

Experimental time series from the computer controlled laboratory experiments on excitable chemical systems with periodic impulse forcing will be analyzed. The singular system analysis (SSA) combined with a low-pass filtering will be used as a self-consistent filtering method for the evaluation of a noisy periodic and aperiodic experimental time series. The transformation matrix arising in the SSA appears in the form appropriate for the computation of the approximations of time derivatives of the signal. The suitable reconstruction space is built form the time derivatives of the time series and the choice of the time delay is not critical when the over-sampling is sufficient. The results of experimental data representation will be used for the construction of low-dimensional mappings ("Phase Transition Curves" and "Phase Excitation Curves"). The results of testing of the above algorithm on iterated experimental mappings and numerical simulations of the low-dimensional mathematical model of the Langevin type will be also presented.

Computer package for chaotic time series analysis

C. Sprott

Computer demonstration.

Noise induced escape across fractal basin boundaries

N.G. Stocks, P.V.E. McClintock, N.D. Stein and R. Mannella

It is well known that nonlinear dissipative systems can have many simultaneously coexisting steady state attractors, each with its own basin of attraction. The boundaries between the basins are often formed by well defined manifolds in phase space, such as the inset of a saddle. The introduction of random fluctuations (noise) into such systems can cause transitions between attractors to occur and, typically, one would expect that for weak noise the mean escape time (MET) across a basin boundary would show an activation type dependence[1] on the noise intensity, D,

$$\mathrm{MET} \propto e^{\frac{\Delta E}{D}} \tag{1}$$

The activation energy, ΔE, is the minimum amount of energy required to force the system over the manifold separating the basins of attraction. In a potential system ΔE would simply represent the potential barrier height between the two attracting states.

We have investigated the dependence of the MET on noise intensity for the system,

$$\ddot{x} + \beta\dot{x} + x - x^2 = F \sin \omega t + \xi(t) \tag{2}$$

where $\xi(t)$ represents Gaussian white noise. This system is studied in a range of parameter space where the basin boundaries are no longer defined by smooth manifolds but are in fact known to be fractal in nature[2]. In this case no well defined escape boundary can exist; in fact one may think of a "spread" of possible escape boundaries. It is however, demonstrated that the MET still shows an activation type dependence on noise intensity. Possible explanations of this surprising result are discussed.

Evaluation of probabilistic and dynamical invariants from finite symbolic substrings - comparison between two approaches

R. Stoop and J. Parisi

We compare the more traditional ensemble approach with the recently proposed cycle expansion technique of periodic orbits. Consequences for the application to experimental time series are outlined.

Singular spectrum analysis, noise filtering, and linear forecasting

R. Vautard, P. Yiou and M. Ghil

We derive the main properties of the Toeplitz matrix associated with the singular spectrum analysis (SSA) for general signals. We emphasize in particular the spectral properties of the eigenfunctions (adaptive filters). One of the main point with SSA is that it usually produces filters involving different frequency bands. These orthogonal filters are used to isolate different components of the signal. Unless there is degeneracy, these components are very weakly coupled. The components can be regular oscillations, in which case they are associated with a pair of nearly-equal eigenvalues. More complex nonlinear behaviour can be found, associated with other groups of eigenvalues.

Besides mathematical interest, SSA has applications in different branches of signal processing. It is shown to be a nice tool for filtering out white noise from time series. A statistical test is derived in order to determine what components, in the tail of the spectrum, are undistinguishable from white noise. The removal of these components filters out the noise in an optimal way. The efficiency of this filtering technique is tested using synthetic noisy series. With a signal-to-noise ratio as low as 0.5, SSA is able to distinguish signal from noise.

Also, the ability of SSA to distinguish more or less periodic components from irregular ones leads to separate the linearly predictable part of a process. When autoregressive models, based on Maximum Entropy estimations, are fitted to the regular components of the signal only, instead of the total signal, the forecast quality is significantly increased, especially in the long range. Again, this idea is tested on synthetic noisy data.

The singular spectrum analysis is applied on the global, yearly averaged, surface temperature record. The data set is 137 years long. The analysis shows periodic activity associated with the El Nino-Southern Oscillation (ENO) in the range 2-6 years, as well as marked 9-year and 21-year periods. There is also a strong irregular trend in the data, associated with the global warming (0.005°C/year, on average). SSA, again, identifies the trend as the fist two principal components. The considerable increase of temperature in the 80's is shown to correspond to superimposition of several oscillations and therefore the detrended part of the signal should decrease in the next few years. Indeed, forecasts are produced that show a deep minimum near 1997.